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1-10	/57	13	/10
11	/15	14	/15
12	/10	Total	/107

MATH 171 Exam 3 Fall 2021

Sections 503 P. Yasskin

Multiple Choice: (5 points each, unless indicated. No part credit.)

1. If $L = \sqrt{x^2 + y^2}$, find $\frac{dL}{dt}$ given that

$$x = 4 \qquad y = 3 \qquad \frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2$$

$\frac{dL}{dt} =$ ____________________

2. Find the horizontal asymptotes of the function $f(x) = \frac{6e^x + 12}{3e^x - 4}$.

a. $y = 2$ only

b. $y = -3$ only

c. $y = \ln\left(\frac{4}{3}\right)$ only

d. $y = 2$ and $y = -3$ only

e. $y = 2$ and $y = \ln\left(\frac{4}{3}\right)$ only

f. $y = -3$ and $y = \ln\left(\frac{4}{3}\right)$ only

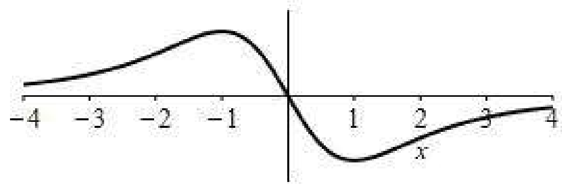
g. $y = 2$, $y = -3$ and $y = \ln\left(\frac{4}{3}\right)$

h. None

3. Find the x-coordinate(s) on the graph of $f(x) = x^4 + 4x^3 + 5x^2 + 3x + 1$ where the **curvature** is a minimum.
Enter one or more numbers separated by commas, no spaces.

$x =$ ____________________

4. (10 points) This is the graph of f',
i.e. the derivative of f.



a. Identify the interval(s) where f is increasing.
Enter one or more intervals separated by commas, no spaces.
Include finite endpoints in the intervals. All numbers are integers. Type infinity for ∞.

Increasing on ______________________________.

b. Identify the interval(s) where f is concave up.
Enter one or more intervals separated by commas, no spaces.
Include finite endpoints in the intervals. All numbers are integers. Type infinity for ∞.

Concave Up on ______________________________.

5. The point $x = 1$ is a critical point of the function $f(x) = x^3 - 3x^2 + 3x$.
Then the Second Derivative Test implies $x = 1$ is a

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. Test Fails

6. (6 points) Find the locations of the absolute maximum and minimum of $f(x) = x^3 - 6x^2 + 9x$ on the interval $[-2, 2]$.

The absolute minimum occurs at $x =$ ____________________.

The absolute maximum occurs at $x =$ ____________________.

7. Use a Riemann Sum with 4 equal intervals and right endpoints to approximate $\int_1^9 (x^2+1)\,dx$.

$\int_1^9 (x^2+1)\,dx \approx$ ____________________

8. (6 points) A rocket starts with an initial height $y(0) = 8$ m and an initial velocity of $v(0) = 0\ \frac{\text{m}}{\text{sec}}$.
If its acceleration is $a(t) = 5e^t$, find its velocity and height at time $t = \ln 3$.
Put an integer in each blank.

$v(\ln 3) =$ _______________ + _______________ $\ln 3$

$y(\ln 3) =$ _______________ + _______________ $\ln 3$

9. Find the area between the curves $y = 3x^2$ and $y = 6x$.

$A =$ ____________________

10. Find the mass of a bar of length π cm, if its linear density is $\delta = 1 + \sin x$ where x is measured from one end.
Put an rational number in each blank.

$M =$ _______________ + _______________ π

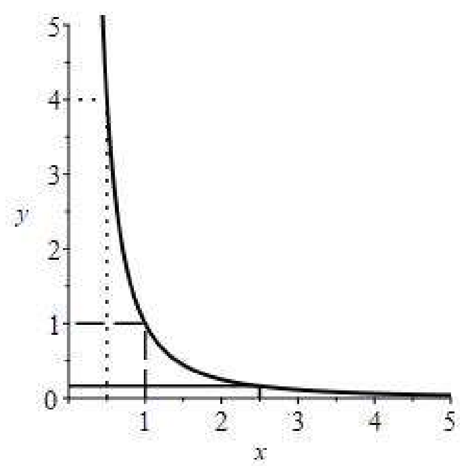
Work Out: (Points indicated. Part credit possible. Show all work.)

11. (15 points) A cone with the vertex at the bottom has height $H = 6$ cm and radius $R = 3$ cm at the top. It is being filled with water at the rate of $\frac{dV}{dt} = 2\pi \frac{\text{cm}^3}{\text{sec}}$.

How fast is the height of the water increasing when it is 4 cm deep?

HINT: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

12. (10 points) Find the perimeter of the rectangle in the first quadrant with the smallest perimeter having one edge on the x-axis and one on the y-axis and the opposite vertex on the curve $y = \frac{4}{x^2}$.



13. (10 points) If $f(x) = \int_{x^2}^{x^3} \frac{1}{t^3+1}\,dt$, find $f'(1)$.

14. (15 points) Compute each integral.

a. $\int x^3 \cos(x^4)\,dx$

b. $\int \frac{x^2+1}{x^3+3x}\,dx$

c. $\int_0^3 2x\sqrt{16+x^2}\,dx$ Simplify to a rational number.