Name_____Section_____

1-7	/70	9	/ 5
8	/10	10	/15
		Total	/100

MATH 171 Exam 1A Fall 2022

Section 502/504 P. Yasskin

Multiple Choice and Short Answer:

(Show your work in case there is part credit.)

1. (5 points) Find the angle between the vectors $\vec{u} = \langle \sqrt{3}, 1 \rangle$ and $\vec{v} = \langle \sqrt{3}, 3 \rangle$.

$\theta =$ _____

2. (10 points) Write $\vec{a} = \langle -1, 7 \rangle$ as the sum of two vectors $\vec{p}$ and $\vec{q}$ where $\vec{p}$ is parallel to $\vec{b} = \langle 8, -6 \rangle$ and $\vec{q}$ is perpendicular to $\vec{b} = \langle 8, -6 \rangle$.

$\vec{a} = \vec{p} + \vec{q}$ where..... $\vec{p} = \langle$_____, _____$\rangle$ and $\vec{q} = \langle$_____, _____$\rangle$

3. (5 points) A line passes through the point $P = (2, -6)$ and is tangent to the direction $\vec{v} = \langle 4, 2 \rangle$. Which of the following points are on the line? (Circle your one answer.)

a. $(6, 2)$

b. $(4, -5)$

c. $(-2, -6)$

d. $(0, -5)$

4. (5 points) Find the part of the real line where the function $f(x) = \sqrt{9 - x^2} + \dfrac{1}{\sqrt{4 - x^2}}$ is continuous.

Continuous on: _____

5. Compute each of the following limits:

a. (5 points) $\lim\limits_{x \to 3} \dfrac{(x+3)^2 - 36}{x - 3}$ ..... = _____

b. (5 points) $\lim\limits_{x \to 2} \dfrac{e^x - e^2}{x - 2}$ ..... = _____

c. (5 points) $\lim\limits_{x \to 4} \dfrac{\sqrt{20 - x} - 4}{x - 4}$ ..... = _____

d. (5 points) $\lim\limits_{x \to -\infty} \dfrac{3 \cdot 2^x + 4 \cdot 3^x}{2^x + 2 \cdot 3^x}$ ..... = _____

6. (5 points) Find the smallest interval with integer endpoints in which there is a solution of the equation $x^5 + 3x^3 = 200$.

There is a solution in the interval $I = [\_\_\_\_\_, \_\_\_\_\_]$.

7. Compute each of the following derivatives:

a. (5 points) $f(x) = 3x^5 + 4x^\pi$ ..... $f'(x) =$ _____

b. (5 points) $g(x) = \dfrac{1}{(\sin x + \cos x)^3}$ ..... $g'(x) =$ _____

c. (5 points) $p(x) = e^\pi + \pi^e$ ..... $p'(x) =$ _____

d. (5 points) Find $f'(1)$, if $f(x) = \dfrac{p(x)\,q(x)}{r(x)}$, given that

$$p(1) = 2, \quad p'(1) = 3, \quad q(1) = 4, \quad q'(1) = 6, \quad r(1) = 4, \quad r'(1) = 2$$

$f'(1) =$ _____

Work Out: (Points indicated. Part credit possible. Show all work.)

8. (10 points) Find the tangent line to the graph of $y = g(x) = \sec x$ at $x = \frac{\pi}{4}$.

$y =$_____

9. (5 points) Use the limit definition of the derivative to prove $\frac{d}{dx}\sin x = \cos x$.

10. (15 points) Prove $\lim_{x\to 4}(7-3x) = -5$ by completing the following three steps.

a. Write out the definition.

$\lim_{x\to 4}(7-3x) = -5$ means:

b. Work backwards to find δ in terms of ε.

c. Complete the proof.