Name_____Section_____

MATH 171   Exam 2A   Fall 2022

Section 502/504   P. Yasskin

Multiple Choice and Short Answer:

(5 points each. Show your work in case there is part credit.)

1-10	/50	13	/10
11	/10	14	/25
12	/10	Total	/105

1. Use the linear approximation to approximate $\sqrt{4.2}$.

    a. 1.949 **f**. 2.02

    b. 1.95 **g**. 2.025

    c. 1.951 **h**. 2.049

    d. 1.975 **i**. 2.05

    e. 1.98 **j**. 2.051

2. Notice that the point $(x,y) = (1,2)$ lies on the curve $3x^4y^3 + 4x^3y^2 = 40$.
What is the slope of the curve, $\dfrac{dy}{dx}$, at $(1,2)$?

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \underline{\hspace{1cm}}$$

3. For the function $f = x^4 - 4x$, the Mean Value Theorem says:
There is a number c in $[2,4]$ where $f'(c) =$

$$f'(c) = \underline{\hspace{1cm}}$$

1

4. If $g(x) = \arctan(x)$, then $g'\left(\frac{3}{4}\right) =$

a. $\frac{3}{4}$ **g.** $\frac{9}{16}$

b. $\frac{4}{3}$ **h.** $\frac{16}{9}$

c. $\frac{3}{5}$ **i.** $\frac{9}{25}$

d. $\frac{5}{3}$ **j.** $\frac{25}{9}$

e. $\frac{4}{5}$ **k.** $\frac{16}{25}$

f. $\frac{5}{4}$ **l.** $\frac{25}{16}$

5. Suppose $f(x) = \frac{1}{x^3}$ and $g(x) = f^{-1}(x)$ is the inverse of $f(x)$. What is $g(8)$?
(This is the function g, not its derivative.)

$g(8) =$ _____

6. Suppose $f(x) = 2x^5 + \frac{1}{x^5}$ and $g(x) = f^{-1}(x)$ is the inverse of $f(x)$. Also notice $f(1) = 3$.
The inverse function theorem allows us to easily compute either $g'(1)$ or $g'(3)$.
Which one and what is its value?

$g'(\underline{\hspace{1cm}}) =$ _____

7. The point $x = 2$ is a critical point of the function $f(x) = x^4 - 8x^3 + 24x^2 - 32x$.
Then the Second Derivative Test says $x = 2$ is a

    a. Local Minimum

    b. Local Maximum

    c. Inflection Point

    d. The Second Derivative Test FAILS.

8. The point $x = 1$ is a critical point of the function $f(x) = x^4 - 4x^3 + 4x^2$.
Then the Second Derivative Test says $x = 1$ is a

    a. Local Minimum

    b. Local Maximum

    c. Inflection Point

    d. The Second Derivative Test FAILS.

9. If $p(t) = \ln(t^4)$, what is $p'(2)$?

$$p'(2) = \underline{\hspace{2cm}}$$

10. If $q(s) = (5 + s^{2/3})^{3/2}$, what is $q'(8)$? (Simplify to a rational number.)

$$q'(8) = \underline{\hspace{2cm}}$$

11. (10 points) A conical cup is filled with water to a height $h = 16$ cm and radius $r = 4$ cm, but it is leaking. If 2 cubic cm leaks out, estimate the change in the height of the water.
(Note: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

$$\Delta h = \underline{\hspace{2cm}}$$

12. (10 points) For a pendulum, the period T and the acceleration of gravity g are related by $T\sqrt{g} = 2\pi\sqrt{L}$ where L is the (constant) length of the pendulum. The pendulum is put on a rocket ship so that g and hence T are changing. When $g = 2\ \dfrac{m}{\sec^2}$ and $T = 1$ sec, what is $\dfrac{dT}{dg}$?
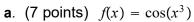
$$\left.\frac{dT}{dg}\right|_{(2,1)} = \underline{\hspace{2cm}}$$

4

13. (10 points)   Find all horizontal and vertical tangents of the parametric curve
$\vec{r}(t) = \left(\frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t, \quad \frac{1}{4}t^4 - \frac{4}{3}t^3 + 2t^2 \right).$

Horizontal tangent(s) at $t =$ _____

Vertical tangent(s) at $t =$ _____

14. (25 points) Find the first and second derivatives of each of the following functions:
(You do not need to simplify, but you may want to simplify the first derivative if it makes it easier to compute the second derivative.)

    a. (7 points) $f(x) = \cos(x^3)$

    $f'(x) = $ _____

    $f''(x) = $ _____

    b. (7 points) $g(x) = \ln(x^2 + 4)$

    $g'(x) = $ _____

    $g''(x) = $ _____

    c. (7 points) $p(x) = \arcsin(4x)$

    $p'(x) = $ _____

    $p''(x) = $ _____

    d. (4 points) $q(x) = x^{(x^3)}$
    HINT: In the base, let $x = e^{(\ln x)}$.
    ON THIS ONE YOU ONLY NEED THE FIRST DERIVATIVE.

    $q'(x) = $ _____