Name_____ Section_____

1-10	/50	13	/10
11	/10	14	/25
12	/10	Total	/105

MATH 171 Exam 2B Fall 2022

Section 502/504 P. Yasskin

Multiple Choice and Short Answer:

(5 points each. Show your work in case there is part credit.)

1. Use the linear approximation to approximate $\sqrt{3.8}$.

a. 1.949
b. 1.95
c. 1.951
d. 1.975
e. 1.98
f. 2.02
g. 2.025
h. 2.049
i. 2.05
j. 2.051

2. Notice that the point $(x,y) = (2,1)$ lies on the curve $3x^3y^4 + 4x^2y^3 = 40.$
What is the slope of the curve, $\frac{dy}{dx}$, at $(2,1)$?

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \_\_\_\_\_$$

3. For the function $f = x^3 - 3x,$ the Mean Value Theorem says:
There is a number c in $[1,3]$ where $f'(c) =$

$$f'(c) = \_\_\_\_\_$$

4. If $g(x) = \arcsin(x)$, then $g'\left(\frac{3}{5}\right) =$

a. $\frac{3}{4}$
b. $\frac{4}{3}$
c. $\frac{3}{5}$
d. $\frac{5}{3}$
e. $\frac{4}{5}$
f. $\frac{5}{4}$
g. $\frac{9}{16}$
h. $\frac{16}{9}$
i. $\frac{9}{25}$
j. $\frac{25}{9}$
k. $\frac{16}{25}$
l. $\frac{25}{16}$

5. Suppose $f(x) = x^5$ and $g(x) = f^{-1}(x)$ is the inverse of $f(x)$. What is $g(32)$? (This is the function g, not its derivative.)

$g(32) =$ _____

6. Suppose $f(x) = 4x^3 + \frac{1}{x^3}$ and $g(x) = f^{-1}(x)$ is the inverse of $f(x)$. Also notice $f(1) = 5$. The inverse function theorem allows us to easily compute either $g'(1)$ or $g'(5)$. Which one and what is its value?

$g'($_____$) =$ _____

7. The point $x = 1$ is a critical point of the function $f(x) = x^4 - 4x^3 + 6x^2 - 4x$.
Then the Second Derivative Test says $x = 1$ is a

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. The Second Derivative Test FAILS.

8. The point $x = 2$ is a critical point of the function $f(x) = x^4 - 4x^3 + 4x^2$.
Then the Second Derivative Test says $x = 2$ is a

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. The Second Derivative Test FAILS.

9. If $p(t) = \ln(t^5)$, what is $p'(10)$?

$p'(10) =$ _____

10. If $q(s) = (2 + s^{1/3})^{3/2}$, what is $q'(8)$? (Simplify to a rational number.)

$q'(8) =$ _____

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (10 points) A conical cup is filled with water to a height $h = 27$ cm and radius $r = 9$ cm, but it is leaking. If 3 cubic cm leaks out, estimate the change in the height of the water.
(Note: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

$\Delta h =$ _____

12. (10 points) A rod is heating up and expanding. The length L and the temperature T are related by $\frac{L - L_0}{T - T_0} = \frac{L_0}{100}$ where $L_0 = 10$ m is the original length and $T_0 = 30°$C is the original temperature. When $L = 12$ m and $T = 50°$C, what is $\frac{dL}{dT}$?

$\left.\frac{dL}{dT}\right|_{(12,50)} =$ _____

13. (10 points) Find all horizontal and vertical tangents of the parametric curve $\vec{r}(t) = \left(\frac{1}{4}t^4 - \frac{4}{3}t^3 + 2t^2, \quad \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t\right)$.

Horizontal tangent(s) at $t =$ _____

Vertical tangent(s) at $t =$ _____

14. (25 points) Find the first and second derivatives of each of the following functions:
(You do not need to simplify, but you may want to simplify the first derivative if it makes it easier to compute the second derivative.)

a. (7 points) $f(x) = \sin(x^4)$

$f'(x) =$ _____

$f''(x) =$ _____

b. (7 points) $g(x) = \ln(x^3 + 6)$

$g'(x) =$ _____

$g''(x) =$ _____

c. (7 points) $p(x) = \arctan(3x)$

$p'(x) =$ _____

$p''(x) =$ _____

d. (4 points) $q(x) = x^{(x^2)}$
HINT: In the base, let $x = e^{(\ln x)}$.
ON THIS ONE YOU ONLY NEED THE FIRST DERIVATIVE.

$q'(x) =$ _____