Name________________________Section____________

MATH 171 Final Exam A Fall 2022

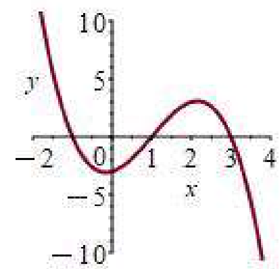
Section 502/504 P. Yasskin

Multiple Choice and Short Answer: Points indicated.

Show your work in case there is part credit.

1-8	/51	11	/12
9	/10	12	/10
10	/10	13	/12
		Total	/105

1. (16 points) Consider a function, $y = f(x)$.
At the right is the graph of its **second** derivative, $y = f''(x)$.
Give answers to the nearest integer.



a. (4 points) Find the interval(s) where $f(x)$ is concave up.

Intervals:____

b. (4 points) Find the location(s) of the inflection point(s) of $f(x)$.

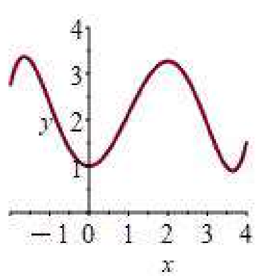
Inflection point(s) at: $x =$____________________________________

c. (4 points) Find the interval(s) where the first derivative $f'(x)$ is decreasing.

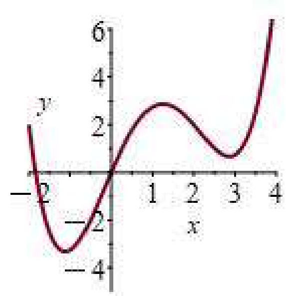
Intervals:____

d. (4 points) Which of these is the graph of $y = f(x)$?

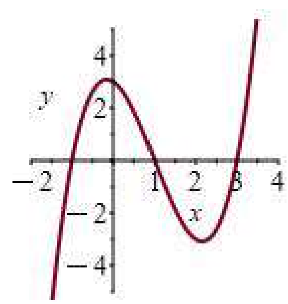
Circle your answer.



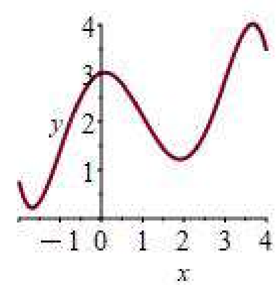
A



B



C



D

2. (5 points) Find the length of the projection of the vector $\vec{u} = \langle 8, -4 \rangle$ onto the vector $\vec{v} = \langle 3, 1 \rangle$.

Length =______________________________

3. (5 points) The point $x = -1$ is a critical point of the function $f(x) = x^4 + \frac{8}{3}x^3 + 2x^2$.
What does the Second Derivative Test say about this point?

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. The test fails

4. (5 points) Find the locations of the global minimum and maximum of the function $f(x) = x^3 - 3x^2 - 9x$ on the interval $[0,4]$.

a. Global Minima at $x =$ ____________________

Global Maxima at $x =$ ____________________

5. (5 points) Find the equation of the line tangent to the curve $x^2y^2 + x^3y^3 = 12$ at $(x,y) = (1,2)$. Then find its y-intercept.

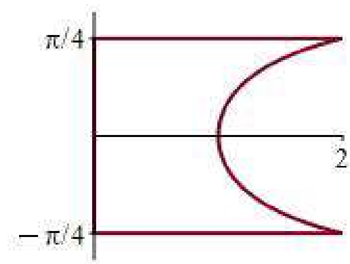
a. Tangent Line: ____________________

y-Intercept $b =$ ____________________

6. (5 points) Find the average value of the function $f(x) = \frac{1}{x}$ on the interval $[1, 4]$. Simplify your answer.

a. $f_{\text{ave}} =$ ______________________________

7. (5 points) Find the area between the graph of $x = \sec^2 y$ and the y-axis for $-\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$. Simplify your answer.



a. $A =$ ______________________________

8. (5 points) Find the mass and center of mass of a 4 cm bar whose linear mass density is $\delta = 12 + 6x$ where x is measured from one end.

a. Mass $M =$ ______________________________

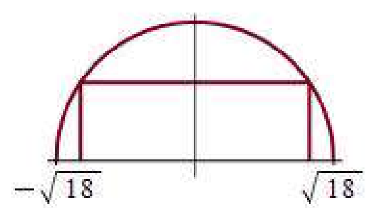
Center of Mass $\bar{x} =$ ______________________________

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (10 points) In an alternate universe, objects fall with constant **jerk** instead of constant acceleration. On planet X, objects fall with constant jerk $j = -12 \frac{\text{m}}{\text{sec}^3}$. If an object is dropped from a height $y = 81$ m with no initial velocity but an initial acceleration of $a(0) = -6 \frac{\text{m}}{\text{sec}^2}$, how long does it take to hit the ground?

Find the time to the nearest integer.

10. (10 points) Find the area of the largest rectangle whose base is on the x-axis and whose upper two vertices are on the semi-circle $y = \sqrt{18 - x^2}$.



11. (12 points) Compute the limits:

a. (6 points) $L = \lim_{x \to 4} \dfrac{\sqrt{5+x} - \sqrt{13-x}}{x-4}$

b. (6 points) $L = \lim_{x \to \infty} (e^x)^{e^{-x}}$

12. (10 points) If $g(x) = \int_{e^{2x}}^{e^{3x}} \dfrac{1}{\ln(t)}\, dt,$ use Leibniz's method to find $g'(x)$ and $g'(1)$.
Simplify exponentials and logs.

13. (12 points) Consider the parametric curve $\vec{r} = (t^2 - 4t + 4, 2t^3 - 9t^2 + 12t + 3)$.

a. (6 points) Find the values of t at all horizontal and vertical tangents to the curve.

Horizontal Tangents at $t =$ ________________________________

Vertical Tangents at $t =$ ________________________________

b. (6 points) Find a parametric tangent line to the curve at $t = 0$.