Name $\qquad$
MATH 172
Exam 2 Spring 2018
Sections 501/502 (circle one)
Multiple Choice: (6 points each. No part credit.)

1. Find the general partial fraction expansion of $f(x)=\frac{(x+2)^{2}}{\left(x^{4}-16\right)(x-2)}$.
a. $\frac{A}{(x-2)^{2}}+\frac{B x+C}{x^{2}+4}$
b. $\frac{A}{x-2}+\frac{B}{(x-2)^{2}}+\frac{C x+D}{x^{2}+4}$
c. $\frac{A}{x-2}+\frac{B x+C}{x^{2}+4}$
d. $\frac{A}{x-2}+\frac{B x+C}{x^{2}+4}+\frac{D x+E}{x^{2}-4}$
e. $\frac{A}{(x-2)^{2}}+\frac{B x+C}{x^{2}+4}+\frac{D x+E}{\left(x^{2}+4\right)^{2}}$
2. Given the partial fraction expansion:

$$
\frac{x^{2}+32 x-4}{x^{4}-16}=\frac{2}{x-2}+\frac{2}{x+2}+\frac{-4 x+1}{x^{2}+4}
$$

which term in the following integral is INCORRECT?

$$
\int \frac{x^{2}+32 x-4}{x^{4}-16} d x=\underbrace{\ln |x-2|^{2}}_{\mathrm{A}}+\underbrace{\ln |x+2|^{2}}_{\mathrm{B}}-\underbrace{\ln \left|x^{2}+4\right|^{2}}_{\mathrm{C}}+\underbrace{\frac{1}{2} \arctan \left(\frac{x}{2}\right)}_{\mathrm{D}}
$$

a. A
b. B
c. C
d. D
e. They are all correct.

| $1-9$ | $/ 54$ | 11 | $/ 15$ |
| ---: | ---: | ---: | ---: |
| 10 | $/ 15$ | 12 | $/ 20$ |
|  |  | Total | $/ 104$ |

3. A spring has a rest length of $x_{0}=5 \mathrm{~m}$. It requires 12 N of force to hold the spring at $x=7 \mathrm{~m}$. Find the work done to stretch the spring from $x=6 \mathrm{~m}$ to $x=8 \mathrm{~m}$.
a. 6
b. 8
c. 12
d. 18
e. 24
4. A 40 ft rope hangs from the top of a building. It's linear weight density is $\rho=3 \mathrm{lb} / \mathrm{ft}$. How much work is done to lift the rope to the top of the building?
a. $2400 \mathrm{ft}-\mathrm{lb}$
b. $1800 \mathrm{ft}-\mathrm{lb}$
c. $1200 \mathrm{ft}-\mathrm{lb}$
d. $600 \mathrm{ft}-\mathrm{lb}$
e. $300 \mathrm{ft}-\mathrm{lb}$
5. The region between $y=\sin x$ and the $x$-axis for $0 \leq x \leq \pi$ is rotated about the $x$-axis. Find the volume swept out.
a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. $\pi^{2}$
d. $\frac{\pi^{2}}{2}$
e. $\frac{\pi^{2}}{4}$
6. The region between $y=10-x^{2}$ and $y=1$ is rotated about the $x$-axis. Which integral gives the volume swept out?
a. $\quad V=2 \pi \int_{0}^{3}\left(9 x-x^{3}\right) d x$
b. $\quad V=\pi \int_{0}^{3}\left(9 x-x^{3}\right) d x$
c. $\quad V=2 \pi \int_{-3}^{3}\left(9 x-x^{3}\right) d x$
d. $\quad V=2 \pi \int_{-3}^{3}\left(x^{4}-20 x^{2}+99\right) d x$
e. $\quad V=\pi \int_{-3}^{3}\left(x^{4}-20 x^{2}+99\right) d x$
7. The region between $y=10-x^{2}$ and $y=1$ is rotated about the $y$-axis.
Find the volume swept out.
a. $\frac{81 \pi}{4}$
b. $\frac{81 \pi}{2}$
c. $18 \pi$
d. $36 \pi$

e. $81 \pi$
8. Compute $\int_{0}^{2} \frac{1}{(x-1)^{2}} d x$.
a. -2
b. 0
c. 2
d. diverges to $+\infty$
e. diverges to $-\infty$
9. Compute $\int_{2}^{\infty} \frac{1}{(x-1)^{2}} d x$.
a. -1
b. 0
c. 1
d. diverges to $+\infty$
e. diverges to $-\infty$

Work Out: (Points indicated. Part credit possible. Show all work.)
10. ( 15 points) The tank shown is 6 m long, 2 m wide at the top and 4 m high. It is filled with water to a depth of 3 m .
How much work is done to pump the water out the top of the tank? Take the density of water to be $\rho \mathrm{kg} / \mathrm{m}^{3}$ and the acceleration of gravity to be $g \mathrm{~m} / \mathrm{sec}^{2}$. (You don't need numbers for $\rho$ and $g$.)

11. (15 points) Find the coefficients in the partial fraction expansion

$$
\frac{10}{\left(x^{2}+4\right)\left(x^{2}-1\right)}=\frac{A x+B}{x^{2}+4}+\frac{C}{x+1}+\frac{D}{x-1}
$$

| $A=$ |
| :--- |
| $B=$ |
| $C=$ |
| $D=$ |

12. (20 points) Use a Comparison Theorem to determine whether each of the following integrals converges or diverges. Clearly state the comparison integral, why the comparison integral converges or diverges and why the original integral converges or diverges.
a. $\int_{1}^{\infty} \frac{1}{\sqrt{x}+x^{2}} d x$
b. $\int_{0}^{1} \frac{1}{\sqrt{x}+x^{2}} d x$
c. What do parts (a) and (b) say about the convergence or divergence of $\int_{0}^{\infty} \frac{1}{\sqrt{x}+x^{2}} d x$ ?
