Name_____

MATH 172 Final Spring 2018

Sections 501/502 (circle one) P. Yasskin

Multiple Choice: (4 points each. No part credit.)

HINTS: $\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$ $\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$

1. $\int_0^{\pi/2} x\cos x\, dx$

a. 1

b. $\frac{\pi}{2}$

c. $1 - \frac{\pi}{2}$

d. $\frac{\pi}{2} - 1$

e. $1 + \frac{\pi}{2}$

2. $\int_0^{\pi/6} \cos^3 x\, dx$

a. $\frac{\pi}{6} - \frac{\pi^3}{3 \cdot 6^3}$

b. $\frac{1}{6}$

c. $\frac{11}{24}$

d. $\frac{3}{8}\sqrt{3}$

e. $\frac{1}{64} - \frac{1}{4}$

1-15	/60	17	/15
16	/10	18	/20
		Total	/105

3. Which coefficient is incorrect in the partial fraction expansion

$$\frac{4}{x^4+4x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$$

a. $A = 0$

b. $B = 1$

c. $C = 0$

d. $D = -1$

e. All coefficients are correct.

4. Find the average value of the function $f = x + \sin^2 x$ on the interval $[0, 2\pi]$.

a. $\pi + \frac{1}{2}$

b. $\pi - \frac{1}{2}$

c. $2\pi^2 + \pi$

d. $2\pi^2 - \pi$

e. $2\pi^2$

5. Find the arclength of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ for $1 \le x \le 3$.

a. 4

b. $\frac{13}{6}$

c. $\frac{13}{3}$

d. $\frac{14}{3}$

e. $\frac{7}{3}$

6. Find the center of mass of an 2 cm bar with density $\rho = x^3$ where x is measured from one end.

a. $\bar{x} = \frac{4}{5}$

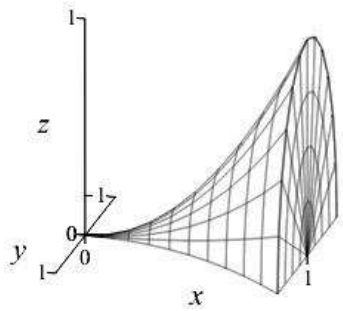
b. $\bar{x} = \frac{8}{5}$

c. $\bar{x} = \frac{32}{5}$

d. $\bar{x} = \frac{5}{4}$

e. $\bar{x} = \frac{5}{8}$

7. Find the volume of a solid whose base is the region between the curves $y = x^2$ and $y = -x^2$ for $0 \le x \le 1$ and whose cross sections perpendicular to the x-axis are semicircles.



a. $\frac{\pi}{6}$

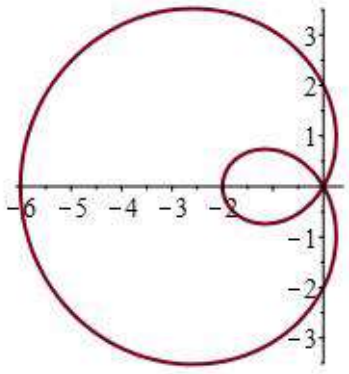
b. $\frac{\pi}{8}$

c. $\frac{\pi}{10}$

d. $\frac{\pi}{12}$

e. $\frac{\pi}{16}$

8. The plot at the right is the graph of which polar function?



a. $r = 2 - 6\cos\theta$

b. $r = -6 + 2\cos\theta$

c. $r = -4 + 2\cos\theta$

d. $r = 4 - 2\cos\theta$

e. $r = 2 - 4\cos\theta$

9. The integral $\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$

a. converges by comparison with $\int_0^1 \frac{1}{x^2}\,dx$

b. diverges by comparison with $\int_0^1 \frac{1}{x^2}\,dx$

c. converges by comparison with $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

d. diverges by comparison with $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

e. diverges by the Divergence Test

10. The series $\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$

a. converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

b. diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

c. converges by comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

d. diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

e. diverges by the Divergence Test

11. $\lim_{n\to\infty}\left(\frac{n^2}{n-1}-\frac{n^2}{n+1}\right)=$

a. -1

b. 0

c. 1

d. 2

e. divergent

12. $S=\sum_{n=1}^{\infty}\left(\frac{n}{n+1}-\frac{n+1}{n+2}\right)=$

a. -1

b. $-\frac{1}{2}$

c. 0

d. $\frac{1}{2}$

e. divergent

13. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{2n}$

a. converges by the Integral Test.

b. diverges because the related absolute series $\sum_{n=1}^{\infty}\frac{n+2}{2n}$ diverges.

c. converges by the Alternating Series Test.

d. diverges by the Alternating Series Test.

e. diverges by the Divergence Test.

14. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n + 3^n}{5^n}(x-4)^n$.

a. $R = \frac{5}{2}$

b. $R = \frac{5}{3}$

c. $R = \frac{2}{5}$

d. $R = \frac{3}{5}$

e. $R = \infty$

15. The series $\sum_{n=0}^{\infty} \frac{1}{3^n\sqrt{n}}(x-5)^n$ has radius of convergence $R = 3$. Find its interval of convergence.

a. $[2,8)$

b. $(2,8]$

c. $[2,8]$

d. $(2,8)$

Work Out: (Points indicated. Part credit possible. Show all work.)

16. $(10$ points$)$ Compute $\int_5^6 \frac{1}{9-x^2}\,dx$. (Also substitutes for Exam 1 #14.)

17. (15 points) The goal is to compute $\lim_{x\to 0} \dfrac{1+x^2-e^{x^2}}{x^4}$.

a. Write out the first 4 terms of the Maclaurin series for e^u.

b. Write out the first 4 terms of the Maclaurin series for e^{x^2}.

c. Substitute the series into $\lim_{x\to 0} \dfrac{1+x^2-e^{x^2}}{x^4}$ and compute the limit.

18. (20 points) The goal is to compute the sum of the series $\sum_{n=0}^{\infty} \frac{n}{2^n}$.

a. Find the sum of the series $\sum_{n=0}^{\infty} x^n$. On what interval does it converge. Why?

$\sum_{n=0}^{\infty} x^n =$ [ ]

Converges for [ ] because…

b. Differentiate both sides of this equation. On what interval does it converge. Why?

$\sum_{n=0}^{\infty} nx^{n-1} =$ [ ]

Converges for [ ] because…

c. Multiply both sides by x. On what interval does it converge. Why?

$\sum_{n=0}^{\infty} nx^n =$ [ ]

Converges for [ ] because…

d. Evaluate both sides at an appropriate value of x and simplify. Why does it converge for this value of x?

$x =$ [ ]: $\sum_{n=0}^{\infty} \frac{n}{2^n} =$ [ ] $=$ [ ]

Converges because…