Name________________________________

MATH 172 Quiz 2 Spring 2018

Sections 501-502 (circle one) P. Yasskin

1	/4	5	/1
2	/5	6	/4
3	/1	7	/1
4	/4	Total	/20

Consider the recursively defined sequence

$$a_1 = 0 \qquad a_n = \sqrt{2 + a_{n-1}}$$

In this quiz, you will prove the sequence a_n converges and find the limit.

1. Write out the first 4 terms of the sequence.

$a_1 =$ ________ $a_2 =$ ____________ $a_3 =$ ____________ $a_4 =$ ____________

2. Assuming the sequence converges, find all possible values for the limit $L = \lim\limits_{n\to\infty} a_n$.

3. Circle a word:

Conjecture: The sequence is increasing / decreasing .

4. Use mathematical induction to prove your conjecture about monotonicity.

a. Check $a_1 \lessgtr a_2 \lessgtr a_3$ with the correct direction of the inequalities.

b. Assume $a_{k-1} \lessgtr a_k$ with the correct inequality and prove $a_k \lessgtr a_{k+1}$ with the same inequality.

5. Circle a word and fill in a number: (Be sure your conjecture will help you prove a_n converges.)

Conjecture: The sequence is bounded above / below by $M =$ ________.

6. Use mathematical induction to prove your conjecture about boundedness.

a. Check $a_1 \lesseqgtr M$ and $a_2 \lesseqgtr M$ with the correct direction of the inequalities.

b. Assume $a_{k-1} \lesseqgtr M$ with the correct inequality and prove $a_k \lesseqgtr M$ with the same inequality.

7. State the version of the Bounded Monotonic Sequence Theorem which guarantees the sequence a_n converges.