1	/4	5	/1
2	/5	6	/4
3	/1	7	/1
4	/4	Total	/20

Name________________________________

MATH 172 Quiz 2 Spring 2018

Sections 501-502 (circle one) Solutions P. Yasskin

Consider the recursively defined sequence

$$a_1 = 0 \qquad a_n = \sqrt{2 + a_{n-1}}$$

In this quiz, you will prove the sequence a_n converges and find the limit.

1. Write out the first 4 terms of the sequence.

Solution: $a_1 = \underline{0}$ $\quad a_2 = \underline{\sqrt{2+0} = \sqrt{2}}$ $\quad a_3 = \underline{\sqrt{2+\sqrt{2}}}$ $\quad a_4 = \underline{\sqrt{2+\sqrt{2+\sqrt{2}}}}$

2. Assuming the sequence converges, find all possible values for the limit $L = \lim\limits_{n\to\infty} a_n$.

Solution: $L = \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \sqrt{2+a_{n-1}} = \sqrt{2 + \lim\limits_{n\to\infty} a_{n-1}} = \sqrt{2+L}$

$L^2 = 2 + L \qquad L^2 - L - 2 = 0 \qquad (L-2)(L+1) = 0 \qquad L = -1, 2$

3. Circle a word:

Conjecture: The sequence is **increasing** (boxed) / decreasing .

4. Use mathematical induction to prove your conjecture about monotonicity.

a. Check $a_1 \lesseqgtr a_2 \lesseqgtr a_3$ with the correct direction of the inequalities.

Solution: $0 < \sqrt{2}$. So $a_1 < a_2$.

$2 < 2 + \sqrt{2}$. So $\sqrt{2} < \sqrt{2+\sqrt{2}}$ or $a_2 < a_3$

b. Assume $a_{k-1} \lesseqgtr a_k$ with the correct inequality and prove $a_k \lesseqgtr a_{k+1}$ with the same inequality.

Solution: Assume $a_{k-1} < a_k$.

Then $2 + a_{k-1} < 2 + a_k$ and $\sqrt{2+a_{k-1}} < \sqrt{2+a_k}$ or $a_k < a_{k+1}$.

5. Circle a word and fill in a number: (Be sure your conjecture will help you prove a_n converges.)

Conjecture: The sequence is bounded [above] / below by $M = 2$.

6. Use mathematical induction to prove your conjecture about boundedness.

a. Check $a_1 \lesseqgtr M$ and $a_2 \lesseqgtr M$ with the correct direction of the inequalities.

Solution: $a_1 = 0 < 2.$ $a_2 = \sqrt{2} < 2.$

b. Assume $a_{k-1} \lesseqgtr M$ with the correct inequality and prove $a_k \lesseqgtr M$ with the same inequality.

Solution: Assume $a_{k-1} < 2.$

Then $2 + a_{k-1} < 4$ and $\sqrt{2 + a_{k-1}} < \sqrt{4} = 2$ or $a_k < 2.$

7. State the version of the Bounded Monotonic Sequence Theorem which guarantees the sequence a_n converges.

Solution: An increasing sequence which is bounded above must converge.