Name________________________________

MATH 172 Exam 3 Spring 2020

Sections 501 P. Yasskin

Multiple Choice: (Points indicated. No part credit.)

1. (1 points) An Aggie does not lie, cheat or steal or tolerate those who do.

True ☐ False ☐

2. (1 points) Each answer is one of the following or a sum of these:

a rational number in lowest terms, e.g. $-\frac{217}{5}$ which is entered as "-217/5"

a rational number in lowest terms times π, e.g. $\frac{217}{5}\pi$ which is entered as "217/5pi"

exponentials such as e^4 or $3^{12/5}$ which are entered as "e^4" or "3^(12/5)"

positive infinity, ∞, which is entered as "infinity"

negative infinity, $-\infty$, which is entered as "-infinity"

convergent, which is entered as "convergent"

divergent, which is entered as "divergent"

Do not leave any spaces. Do not use decimals.

I read this.

True ☐ False ☐

3. (4 points) Compute $\sum_{n=1}^{\infty} \frac{4}{2^n}$.

a. 0

b. 1

c. 2

d. 4

e. ∞

1-9	/30	13	/18
10-11	/28	14	/8
12	/18	Total	/102

4. (4 points) Compute $\sum_{n=1}^{\infty}\left(\frac{n}{2n-1}-\frac{n+1}{2n+1}\right)$.

a. 0

b. $\frac{1}{2}$

c. 1

d. $\frac{3}{2}$

e. ∞

5. (4 points) Compute $\lim_{n\to\infty}\left(\sqrt{n^6+5n^3}-\sqrt{n^6-4n^3}\right)$.

a. $-\infty$

b. -4

c. 9

d. $\frac{9}{2}$

e. ∞

6. (4 points) Compute $\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{4n}$. If divergent, enter "infinity" or "-infinity".

a. e

b. e^2

c. e^4

d. e^8

e. ∞

7. (4 points) Compute $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$. If divergent, enter "infinity" or "-infinity".

a. e^3

b. e^{-3}

c. $-e^3 - 1$

d. $e^{-3} - 1$

e. ∞

8. (4 points) If $S = \sum_{n=1}^{\infty} a_n$ and $S_k = \frac{k}{2k+1}$, then

a. $S = 0$

b. $S = 1$

c. $S = \frac{1}{2}$

d. $S = \frac{1}{3}$

e. $S = \infty$

9. (4 points) If the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by the 99^{th} partial sum $S_{99} = \sum_{n=1}^{99} \frac{(-1)^{n+1}}{n^3} \approx 0.90154318486844623867$, how many digits of accuracy are guaranteed in this approximation? For example, if the error is $|E_{99}| < 10^{-5}$, then only the digits 0.9015 are accurate, and you would answer 4.

a. 4

b. 5

c. 10

d. 100

e. 1000000

10. (14 points) The series $\sum_{n=2}^{\infty} \frac{1}{n-1}$ can be shown to diverge by which of the following Convergence Tests? Check Yes for all that work; check No for all that don't work.

a. n^{th}-Term test for Divergence:

☐ Yes ☐ No

b. Integral Test:

☐ Yes ☐ No

c. p-Series Test:

☐ Yes ☐ No

d. Simple Comparison Test comparing to $\sum_{n=2}^{\infty} \frac{1}{n}$:

☐ Yes ☐ No

e. Limit Comparison Test comparing to $\sum_{n=2}^{\infty} \frac{1}{n}$:

☐ Yes ☐ No

f. Ratio Test:

☐ Yes ☐ No

g. Alternating Series Test:

☐ Yes ☐ No

11. (14 points) The series $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ can be shown to converge by which of the following Convergence Tests? Check Yes for all that work; check No for all that don't work.

a. n^{th}-Term test for Divergence:

☐ Yes ☐ No

b. Integral Test:

☐ Yes ☐ No

c. p-Series Test:

☐ Yes ☐ No

d. Simple Comparison Test comparing to $\sum_{n=2}^{\infty} \frac{1}{n^2}$:

☐ Yes ☐ No

e. Limit Comparison Test comparing to $\sum_{n=2}^{\infty} \frac{1}{n^2}$:

☐ Yes ☐ No

f. Ratio Test:

☐ Yes ☐ No

g. Alternating Series Test:

☐ Yes ☐ No

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (18 points) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{1+\sqrt{n}}(x-3)^n$.

a. Find the radius of convergence and state the open interval of absolute convergence.

$R =$ ____. Absolutely convergent on (_____,_____).

b. Check the **Left** Endpoint:

$x =$_____ The series becomes_________________________ Circle one:

Reasons: Convergent

Divergent

c. Check the **Right** Endpoint:

$x =$_____ The series becomes_________________________ Circle one:

Reasons: Convergent

Divergent

d. State the Interval of Convergence. Interval= ____________

13. $\left(18 \text{ points}\right)$ Determine whether the recursively defined sequence $a_1 = 2\sqrt{6}$ and $a_{n+1} = \dfrac{(a_n)^2 + 16}{10}$ is convergent or divergent. If convergent, find the limit. If divergent, say infinity or -infinity.

a. Find the first 3 terms: $a_1 =$ ______ $a_2 =$ ______ $a_3 =$ ______

b. Assuming the limit $\lim\limits_{n \to \infty} a_n$ exists, find the possible limits.

c. Prove the sequence is bounded or unbounded above or below (as appropriate).

d. Prove the sequence is increasing or decreasing (as appropriate).

e. State whether the sequence is convergent or divergent and name the theorem. If convergent, determine the limit. If divergent, determine if it is infinity or -infinity.

14. $(8$ points$)$ A ball is dropped from a height of 72 feet. Each time it bounces it reaches a height which is $\frac{1}{2}$ of the height on the previous bounce. What is the total distance travelled by the ball (with an infinite number of bounces)?