

Name _____

MATH 172

Exam 3

Spring 2023

Sections 502

P. Yasskin

1-7	/35	13	/20
8-12	/50	Total	/105

Multiple Choice: (5 points each. No part credit. Circle your answers.)

1. $L = \lim_{n \rightarrow \infty} \frac{\frac{4}{n^2} - \frac{9}{n^3}}{\frac{2}{n^2} + \frac{3}{n^3}} =$

- a. -3
- b. 0
- c. 2
- d. 3
- e. diverges

2. $L = \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} \right)^{\ln n} =$

- a. -4
- b. $\ln 4$
- c. $-\ln 4$
- d. e^{-4}
- e. diverges

3. $L = \lim_{n \rightarrow \infty} n^2 \left[1 - \cos\left(\frac{1}{n}\right) \right] =$

- a. $\frac{1}{2}$
- b. 2
- c. -2
- d. $-\frac{1}{2}$
- e. diverges

4. $\sum_{n=2}^{\infty} \frac{(-2)^n}{3^n} =$

a. $\frac{4}{15}$

b. $\frac{3}{5}$

c. $\frac{4}{3}$

d. $-\frac{2}{5}$

e. divergent

5. $\sum_{n=2}^{\infty} \frac{2^{2n}}{3^n} =$

a. $-\frac{16}{3}$

b. $\frac{4}{7}$

c. $\frac{16}{3}$

d. $\frac{16}{21}$

e. divergent

6. $\sum_{n=1}^{\infty} [\arctan(n) - \arctan(n+1)] =$

a. $-\frac{\pi}{2}$

b. $-\frac{\pi}{4}$

c. 0

d. $\frac{\pi}{4}$

e. $\frac{\pi}{2}$

7. If $S = \sum_{n=2}^{\infty} a_n$ and $S_k = \sum_{n=2}^k a_n = \frac{k-1}{k}$, what is a_k ?

HINT: What are S_{k-1} and $S_k - S_{k-1}$?

- a. $a_k = \frac{k}{k-1}$
- b. $a_k = \frac{k-1}{k}$
- c. $a_k = \frac{k}{k+1}$
- d. $a_k = \frac{1}{k(k+1)}$
- e. $a_k = \frac{1}{k(k-1)}$

Short Answer: (10 points each. No part credit. Circle your answers.)

For each series, circle **Convergent** or **Divergent** and circle **the test you used**.

(Most series have more than one acceptable test. **Only circle one!**)

For a Comparison or Absolute Convergence Test, write the Comparison series or the Related Absolute series and the write the name of the test you used for that series.

For a Divergence or Integral or Limit Comparison or Ratio Test, compute the required limit or integral. For a Simple Comparison Test, check the required inequality.

8. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n^{3/2} - 1}$ Convergent Divergent

- a. Geometric Series
- b. Telescoping Series
- c. n^{th} Term Divergence Test
- d. Integral Test
- e. p -Series Test
- f. Simple Comparison Test
- g. Limit Comparison Test
- h. Alternating Series Test
- i. Absolute Convergence Test
- j. Ratio Test

Comparison or Absolute Series _____ Its test: _____

Any required limit or integral or inequality:

9. $\sum_{n=0}^{\infty} \frac{\arctan n}{1+n^2}$ Convergent Divergent

- | | |
|---|------------------------------|
| a. Geometric Series | f. Simple Comparison Test |
| b. Telescoping Series | g. Limit Comparison Test |
| c. n^{th} Term Divergence Test | h. Alternating Series Test |
| d. Integral Test | i. Absolute Convergence Test |
| e. p -Series Test | j. Ratio Test |

Comparison or Absolute Series _____ Its test: _____

Any required limit or integral or inequality:

10. $\sum_{n=1}^{\infty} n!e^{-n}$ Convergent Divergent

- | | |
|---|------------------------------|
| a. Geometric Series | f. Simple Comparison Test |
| b. Telescoping Series | g. Limit Comparison Test |
| c. n^{th} Term Divergence Test | h. Alternating Series Test |
| d. Integral Test | i. Absolute Convergence Test |
| e. p -Series Test | j. Ratio Test |

Comparison or Absolute Series _____ Its test: _____

Any required limit or integral or inequality:

11. $\sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$

Convergent Divergent

- | | |
|---|------------------------------|
| a. Geometric Series | f. Simple Comparison Test |
| b. Telescoping Series | g. Limit Comparison Test |
| c. n^{th} Term Divergence Test | h. Alternating Series Test |
| d. Integral Test | i. Absolute Convergence Test |
| e. p -Series Test | j. Ratio Test |

Comparison or Absolute Series _____ Its test: _____

Any required limit or integral or inequality:

12. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^3}$

Convergent Divergent

- | | |
|---|------------------------------|
| a. Geometric Series | f. Simple Comparison Test |
| b. Telescoping Series | g. Limit Comparison Test |
| c. n^{th} Term Divergence Test | h. Alternating Series Test |
| d. Integral Test | i. Absolute Convergence Test |
| e. p -Series Test | j. Ratio Test |

Comparison or Absolute Series _____ Its test: _____

Any required limit or integral or inequality:

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (20 points) Consider the recursively defined sequence $a_{n+1} = 5 - \frac{4}{a_n}$ with $a_1 = 2$

a. (5 pts) Assuming the limit exists, find the possible values of the limit.

b. (2 pts) (Fill in the blanks.) Compute the first 3 terms of the sequence:

$$a_1 = \underline{\hspace{2cm}} \quad a_2 = \underline{\hspace{2cm}} \quad a_3 = \underline{\hspace{2cm}}$$

c. (2 pt) (Fill in the blanks.) The sequence appears to be bounded

below by $\underline{\hspace{2cm}}$ and above by $\underline{\hspace{2cm}}$.

d. (4 pts) Use induction to prove the sequence is bounded below and above by your answers.

e. (1 pt) (Circle one.) The sequence appears to be increasing decreasing.

f. (4 pts) Use induction to prove the sequence is increasing or decreasing.

g. (2 pt) What do you conclude about the limit of the sequence? Why?
What theorem are you using? (This must be sentences.).