Name_____

MATH 172H Exam 3 Spring 2019

Sections 200 P. Yasskin

15 Multiple Choice: (4 points each. No part credit.)

1. Compute $\lim_{n\to\infty} \frac{(-2)^n - (-3)^n}{(-3)^n}$.

   a. -2
   b. -1
   c. 1
   d. 2
   e. diverges

2. Compute $\lim_{n\to\infty} \left(\sqrt{n^4 + 4n^2} - \sqrt{n^4 - 2n^2} \right)$.

   a. $-\infty$
   b. -6
   c. 3
   d. 6
   e. ∞

1-15	/60	17	/15
16	/10	18	/20
		Total	/105

3. Compute $\lim_{n\to\infty}\left(1-\frac{2}{n^2}\right)^n$

a. 0
b. e^{-4}
c. e^{-2}
d. e^{-1}
e. 1

4. If $S=\sum_{n=1}^{\infty}a_n$ and $S_k=\frac{k}{k+1}$, then

a. $a_n=\frac{-1}{n(n+1)}$
b. $a_n=\frac{1}{n(n-1)}$
c. $a_n=\frac{2}{n(n-1)}$
d. $a_n=\frac{1}{n(n+1)}$
e. $a_n=\frac{2}{n(n+1)}$

5. Compute $\sum_{n=2}^{\infty}\left(\frac{2n+1}{n}-\frac{2n+3}{n+1}\right)$

a. $\frac{1}{2}$
b. $\frac{3}{2}$
c. 2
d. –2
e. 0

6. For this and the next problem, consider the series $\sum_{n=0}^{\infty} \frac{1}{e^n + 1}$. This series

   a. converges to a number less than e^{-1}
   b. converges to a number less than $\frac{e}{e-1}$
   c. converges to a number greater than $\frac{e}{e-1}$
   d. diverges to ∞
   e. diverges but not to ∞

7. Which test did you use in the previous problem?

   a. Integral Test
   b. Simple Comparison Test
   c. Limit (but not Simple) Comparison Test
   d. Alternating Series Test
   e. n^{th} Term Divergence Test

8. The series $\sum_{n=1}^{\infty} \frac{2n+2}{n^2+2n}$

   a. converges by the Integral Test
   b. diverges by the Integral Test
   c. converges by a Simple Comparison with $\sum_{n=1}^{\infty} \frac{2}{n^2}$
   d. diverges by a Simple Comparison with $\sum_{n=1}^{\infty} \frac{2}{n}$
   e. converges by the Ratio Test

9. The series $S = \sum_{n=1}^{\infty} \frac{2n+2}{(n^2+2n)^2}$ converges by the Integral Test. If we approximate S by $S_{10} = \sum_{n=1}^{10} \frac{2n+2}{(n^2+2n)^2}$, find a bound on the error $E_{10} = S - S_{10} = \sum_{n=11}^{\infty} \frac{2n+2}{(n^2+2n)^2}$.

a. $|E_{10}| < \frac{1}{120}$
b. $|E_{10}| < \frac{1}{143}$
c. $|E_{10}| < \frac{1}{150}$
d. $|E_{10}| < \frac{1}{160}$
e. $|E_{10}| < \frac{1}{180}$

10. For this and the next problem, consider the series $\sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}}$. This series

a. converges
b. diverges to ∞
c. diverges to $-\infty$
d. diverges but not to $\pm\infty$

11. Which test did you use in the previous problem?

a. Integral Test
b. Simple Comparison Test
c. Limit Comparison Test but not the Simple Comparison Test
d. Alternating Series Test
e. n^{th} Term Divergence Test

12. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ is

a. absolutely convergent
b. conditionally convergent
c. divergent
d. conditionally divergent

13. The series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ is convergent by

a. the Alternating Series Test
b. the Related Absolute Series Test, the Simple Comparison Test and the *p*-Series Test
c. the Related Absolute Series Test, the Limit (but not Simple) Comparison Test and the *p*-Series Test
d. the n^{th} Term Divergence Test

14. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{3n+2}{(-4)^n}(x-2)^n$

a. $R = \infty$
b. $R = 3$
c. $R = 4$
d. $R = \frac{1}{3}$
e. $R = \frac{1}{4}$

15. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(2n+1)!}{3^n}(x-5)^n$

a. $R = \infty$
b. $R = 3$
c. $R = 5$
d. $R = \frac{1}{3}$
e. $R = 0$

Work Out: (Points indicated. Part credit possible. Show all work.)

16. (10 points) Prove $\lim_{n\to\infty} \frac{1}{n^3} = 0.$

a. Write out the $\varepsilon - N$ definition of this limit.

b. Given an ε, what N should you use?

c. Complete the proof.

17. (15 points) Determine whether the recursively defined sequence $a_1 = 4$ and $a_{n+1} = 3\sqrt{a_n}$ is convergent or divergent. If convergent, find the limit.

a. Find the first 3 terms: $a_1 =$ _____ $a_2 =$ _____ $a_3 =$ _____

b. Assuming the limit $\lim_{n\to\infty} a_n$ exists, find the possible limits.

c. Prove the sequence is increasing or decreasing (as appropriate).

d. Prove the sequence is bounded or unbounded above or below (as appropriate).

e. State whether the sequence is convergent or divergent and name the theorem. If convergent, state the limit.

18. (20 points) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+1)3^n}(x-5)^n$.

a. Find the radius of convergence and state the open interval of absolute convergence.

$R =$ ____. Absolutely convergent on (_____,_____).

b. Check the **Left** Endpoint:

$x =$_____ The series is_____ Circle one:

Reasons: Convergent

Divergent

c. Check the **Right** Endpoint:

$x =$_____ The series is_____ Circle one:

Reasons: Convergent

Divergent

d. State the Interval of Convergence. Interval= _____