Name________________________________

MATH 172H Final Exam Spring 2019

Sections 200 P. Yasskin

15 Multiple Choice: (4 points each. No part credit.)

1. Compute $\int_1^e \frac{\ln x}{x^2}\,dx$.

a. 1

b. $1-\frac{2}{e}$

c. $-1-\frac{2}{e}$

d. $\frac{2}{e}-1$

e. $1+\frac{2}{e}$

2. Compute $\int \frac{\sqrt{x^2-4}}{x}\,dx$.

a. $\sqrt{x^2-4}-2\operatorname{arcsec}\frac{x}{2}+C$

b. $2\operatorname{arcsec}\frac{x}{2}-\sqrt{x^2-4}+C$

c. $\frac{\sqrt{x^2-4}}{2}-\ln\left(\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right)+C$

d. $\ln\left(\frac{x}{2}+\frac{\sqrt{x^2-4}}{2}\right)-\frac{\sqrt{x^2-4}}{2}+C$

e. $\ln\left(x+\sqrt{x^2-4}\right)-\sqrt{x^2-4}+C$

1-15	/60	18	/15
16	/10	19	/10
17	/10	Total	/105

3. The integral $\int_1^\infty \frac{1}{x^3 + \sqrt[3]{x}}\,dx$

a. converges by comparison to $\int_1^\infty \frac{1}{x^3}\,dx$.

b. diverges by comparison to $\int_1^\infty \frac{1}{x^3}\,dx$.

c. converges by comparison to $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$.

d. diverges by comparison to $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$.

4. Find the average value of the function $f(x) = \frac{1}{4+x^2}$ on the interval $[0,2]$.

a. 2π

b. $\frac{\pi}{2}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{8}$

e. $\frac{\pi}{16}$

5. Find the center of mass of a bar which is 4 cm long and has density $\delta = 3x + 2x^2$ where x is measured from one end.

a. $\frac{17}{24}$

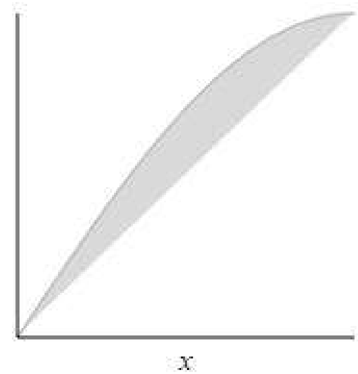
b. $\frac{24}{17}$

c. $\frac{25}{72}$

d. $\frac{72}{25}$

e. $\frac{200}{3}$

6. The region between $y = \sin x$ and $y = \frac{2x}{\pi}$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis. Which integral gives the volume swept out?



a. $V = \int_0^{\pi/2} 2\pi x\left(\frac{2x}{\pi} - \sin x\right) dx$

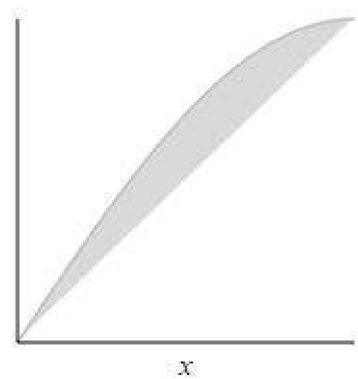
b. $V = \int_0^{\pi/2} 2\pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

c. $V = \int_0^{\pi/2} 2\pi x\left(\sin x - \frac{2x}{\pi}\right) dx$

d. $V = \int_0^{\pi/2} \pi\left(\frac{4x^2}{\pi^2} - \sin^2 x\right) dx$

e. $V = \int_0^{\pi/2} \pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

7. The region between $y = \sin x$ and $y = \frac{2x}{\pi}$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the x-axis. Which integral gives the volume swept out?



a. $V = \int_0^{\pi/2} 2\pi x\left(\frac{2x}{\pi} - \sin x\right) dx$

b. $V = \int_0^{\pi/2} 2\pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

c. $V = \int_0^{\pi/2} 2\pi x\left(\sin x - \frac{2x}{\pi}\right) dx$

d. $V = \int_0^{\pi/2} \pi\left(\frac{4x^2}{\pi^2} - \sin^2 x\right) dx$

e. $V = \int_0^{\pi/2} \pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

8. Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ with the initial condition $y(1) = 2$. Then $y(3) =$

a. -6
b. 4
c. $\sqrt[3]{7}$
d. $\sqrt[3]{20}$
e. $\sqrt[3]{34}$

9. For the differential equation, $\frac{1}{y}\frac{dy}{dx} = \frac{5}{xy} + \frac{3}{x}$, the integrating factor is

a. $-3\ln x$
b. $\frac{1}{x^3}$
c. $-\frac{3}{x}$
d. x^3
e. There is no integrating factor since the equation is not linear.

10. Compute $L = \lim_{n\to\infty}\left(\frac{n-3}{n-1}\right)^n$.

a. -2
b. e^{-2}
c. 1
d. e^2
e. ∞

11. A sequence a_n is defined recursively by $a_{n+1} = \frac{(a_n)^2 + 2}{3}$ and $a_1 = 4$. Find $\lim_{n\to\infty} a_n$.

a. 1
b. 2
c. 4
d. 16
e. ∞

12. The series $\sum_{n=2}^{\infty} \frac{3n}{n^3 - 2}$

a. converges by Simple Comparison with $\sum_{n=2}^{\infty} \frac{3}{n^2}$.

b. diverges by Simple Comparison with $\sum_{n=2}^{\infty} \frac{3}{n^2}$.

c. converges by Limit but not Simple Comparison with $\sum_{n=2}^{\infty} \frac{3}{n^2}$.

d. diverges by Limit but not Simple Comparison with $\sum_{n=2}^{\infty} \frac{3}{n^2}$.

e. converges by the Ratio Test.

13. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (x - 3)^n$.

a. 0
b. 1
c. 2
d. 4
e. ∞

14. The series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n (n^3 + \sqrt[3]{n})}$ has radius of convergence $R = 3$. Find its interval of convergence.

a. $(-1, 5)$
b. $[-1, 5)$
c. $(-1, 5]$
d. $[-1, 5]$

15. For the function $f(x) = \cos(x^3)$ which of the following is FALSE?.

a. $f^{(27)}(0) = 0$
b. $f^{(28)}(0) = 0$
c. $f^{(29)}(0) = 0$
d. $f^{(30)}(0) = 0$
e. $f^{(31)}(0) = 0$

Work Out: (Points indicated. Part credit possible. Show all work.)

16. (10 points) Compute $\int \frac{2}{x^3 - x}\, dx.$

a. Find the general partial fraction expansion. (Do not find the coefficients.)

$$\frac{2}{x^3 - x} = \underline{\hspace{6cm}}$$

b. Find the coefficients and plug them back into the expansion.

$$\frac{2}{x^3 - x} = \underline{\hspace{6cm}}$$

c. Compute the integral.

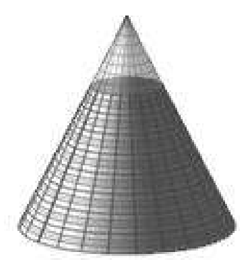
$$\int \frac{2}{x^3 - x}\, dx = \underline{\hspace{6cm}}$$

17. (10 points) The curve $(x,y) = \left(\frac{1}{2}t^2, \frac{1}{3}t^3\right)$ for $0 \le t \le \sqrt{3}$ is rotated about the y-axis. Find the area of the surface swept out.

18. (15 points) A water tank has the shape of a cone with the vertex at the top. Its height is $H = 30$ ft and its radius is $R = 10$ ft.

It is filled with salt water to a depth of 20 ft which weighs $\delta = 63 \dfrac{\text{lb}}{\text{ft}^3}$.

Find the work done to pump the water out the top of the tank.



19. (10 points) The Maclaurin series for $f(x) = \dfrac{1}{1-x}$ is the geometric series $\sum_{n=0}^{\infty} x^n$ which converges for $|x| < 1$. For $x < 0$, the series is alternating; for $x > 0$ it is positive. We will approximate the series on the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ by the 9^{th} degree Maclaurin polynomial which is the 9^{th} partial sum $S_9(x) = \sum_{n=0}^{9} x^n$. The error in this approximation is the remainder $R_9(x) = f(x) - S_9(x)$, which of course depends on the value of x.

a. Find the Alternating Series bound on the remainder for $x \in \left(-\frac{1}{2}, 0\right)$.
NOTE: This should be a single number which works for all values of x in the interval.

b. The Taylor Remainder Inequality says

$$|R_n(x)| < \frac{M}{(n+1)!}|x|^{n+1} \qquad \text{where } \dot{M} \geq f^{(n+1)}(c) \text{ for all } c \text{ between } 0 \text{ and } x.$$

Find the Taylor Remainder bound on the remainder for $x \in \left(0, \frac{1}{2}\right)$.
NOTE: This should be a single number which works for all values of x in the interval.