# M172-19c, EXAMINATION I

*Show all your work clearly; unsupported answers will be eyed with suspicion, and will be denied credit. As sacrilegious as it may be, calculators may not be used, whilst cell phones must be turned off. Untidy work will be treated with the utmost disdain it deserves.*

**Total marks: 106**

**1.** Suppose that $f$ is a bounded real-valued function defined on the interval $[a, b]$.

(i) (`3 marks`) Define the terms 'partition of $[a, b]$', 'norm of a partition', and 'Riemann sum of $f$ associated to a partition'.

(ii) (`3 marks`) Define what it means to say that $f$ is Riemann integrable on $[a, b]$.

**2.** (`6 marks`) State (both parts of) the Fundamental Theorem of Calculus.

**3.** (`3 marks`) State the Theorem on Integration by Parts.

**4.** (i) (`3 marks`) State the Mean-Value Theorem for Integrals.

(ii) (`4 marks`) Show that there is a number $c$ in the interval $[1, 9]$ such that
$\int_1^3 t^2 \sin(t^2)\, dt = 4\sqrt{c}\, \sin(c)$.

**5.** (`7 marks`) Evaluate

$$\lim_{x\to 1}\left[\frac{\int_{\sin(\pi x)}^{x(x-1)} \sqrt{1+t^4}\, dt}{x^2+x-2}\right].$$

**6.** (`7 marks`) Evaluate $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2+k^2}$.

**7.** (7 `marks`) Compute $\displaystyle\int \frac{x^3}{(x^2+1)^{1/3}}\,dx$.

**8.** (7 `marks`) Compute the area of the region enclosed, in the first quadrant, by $y = 2x^2$, $y = \dfrac{x^2}{4}$, and $x + y = 3$.

**9.** Let $C$ denote the curve $y = e^x$, and let $L$ denote the tangent to $C$ at the point $(a, e^a)$, where $a > 0$.

(i) (7 `marks`) Obtain the value of $a$, and hence an equation for $L$, given that $L$ passes through the origin.

(ii) (7 `marks`) Let $\mathcal{R}$ denote the region enclosed, in the first quadrant, by the $y$-axis, $C$, and $L$. Use the method of washers to compute the volume of the solid obtained by rotating $\mathcal{R}$ about the $x$-axis.

(iii) (7 `marks`) Let $a$ be the value found in (i), and let $\mathcal{R}$ be as above. Use the method of shells to calculate the volume of the solid generated by revolving $\mathcal{R}$ about the line $x = a$.

**10.** (7 `marks`) Suppose that $f$ is a continuous function, and that, *for every* $s > 0$, the average value of $f$ over the interval $[0, s]$ is $\arctan(s)$. Find $f(1)$.

**11.** (7 `marks`) Evaluate $\displaystyle\int_1^e x^n \ln(x)\,dx$, where $n$ is a fixed (but arbitrary) positive integer.

**12.** (7 `marks`) Evaluate $\displaystyle\int_0^{\pi/2} \sin^8(x)\cos^2(x)\,dx$.

**13.** (7 `marks`) Evaluate $\displaystyle\int_0^1 \tan^3(x)\sec(x)\,dx$.

**14.** (7 `marks`) Compute $\displaystyle\int \sin(7x)\cos(5x)\sin(3x)\,dx$.