Name________________________________

MATH 172H Final Exam Fall 2019

Sections 202 P. Yasskin

1-13	/65	15	/20
14	/10	16	/10
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^{\pi/4} \cos\theta \sin^3\theta \, d\theta$

a. $\dfrac{1}{2}$

b. $\dfrac{1}{4}$

c. $\dfrac{1}{8}$

d. $\dfrac{1}{16}$

e. $\dfrac{1}{32}$

2. Compute $\int_0^{\ln 2} xe^{-x} \, dx$

a. $\dfrac{1}{2}\ln 2 + \dfrac{1}{2}$

b. $-\dfrac{1}{2}\ln 2 + \dfrac{1}{2}$

c. $\dfrac{1}{2}\ln 2 - \dfrac{1}{2}$

d. $-\dfrac{1}{2}\ln 2 - \dfrac{1}{2}$

e. Divergent

3. Find the average value of $f(x) = \cos x$ on the interval $-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$.

a. $\dfrac{2\sqrt{2}}{\pi}$

b. $\dfrac{\sqrt{2}}{\pi}$

c. $\sqrt{2}$

d. $\dfrac{1}{\sqrt{2}}$

e. $\dfrac{\pi}{\sqrt{2}}$

4. The partial fraction decomposition of $\frac{1}{x^2 - x}$ is

a. $\frac{1}{x-1} + \frac{1}{x}$

b. $\frac{1}{x-1} - \frac{1}{x}$

c. $\frac{1}{x} - \frac{1}{x-1}$

d. $\frac{1}{x} + \frac{1}{x+1}$

e. $\frac{1}{x+1} - \frac{1}{x}$

5. Compute $\int_3^{3\sqrt{2}} \frac{\sqrt{x^2 - 9}}{x}\, dx$.

a. $\frac{1}{\sqrt{2}} - 1$

b. $1 - \frac{1}{\sqrt{2}}$

c. $3\left(1 - \frac{\pi}{4}\right)$

d. $3\left(\frac{\pi}{4} - 1\right)$

e. ∞

6. Find the arc length of the curve $y = \frac{x^2}{4} - \frac{\ln x}{2}$ between $x = 1$ and $x = e$.

a. $\frac{e^2}{4} - \frac{1}{2}$

b. $\frac{e^2}{2} - \frac{1}{2}$

c. $\frac{e^2}{2} + \frac{1}{2}$

d. $\frac{e^2}{4} - \frac{1}{4}$

e. $\frac{e^2}{4} + \frac{1}{4}$

7. The base of a solid is the semi-circle between $y = \sqrt{9 - x^2}$ and the x-axis.
The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

a. $\frac{9}{2}\pi$

b. 9π

c. 9

d. 18

e. 36

8. The region bounded by $x = 0,\quad x = \cos y,\quad y = 0,\quad y = \frac{\pi}{4}$ is rotated about the x-axis.
Which integral gives the volume of the solid of revolution?

a. $\int_0^{\pi/4} 2\pi \cos^2 y \, dy$

b. $\int_0^{\sqrt{2}/2} 2\pi x \arccos x \, dx$

c. $\int_0^{\pi/4} 2\pi y \cos y \, dy$

d. $\int_0^{\sqrt{2}/2} \pi(\cos^2 x - x^2)\, dx$

e. $\int_0^{\pi/4} 2\pi y^2 \, dy$

9. As n approaches infinity, the sequence $a_n = \frac{1 - \cos n}{n^2}$

a. converges to $-\frac{1}{2}$

b. converges to 0

c. converges to $\frac{1}{2}$

d. converges to 1

e. diverges

10. $\sum_{n=2}^{\infty} \frac{3^n}{4^{n-1}} =$

a. $\frac{9}{7}$

b. 3

c. 4

d. 9

e. Diverges

11. Compute $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} - \frac{n+1}{n+2} \right)$

a. $-\frac{1}{2}$

b. $\frac{1}{2}$

c. 1

d. 2

e. Divergent

12. Compute $\lim_{x \to 0} \frac{\sin(2x) - 2x}{3x^3}$

a. $-\frac{1}{9}$

b. -4

c. $-\frac{8}{9}$

d. $-\frac{4}{3}$

e. $-\frac{4}{9}$

13. If $g(x) = \cos(x^2)$, what is $g^{(8)}(0)$, the 8^{th} derivative at zero?
HINT: What is the coefficient of x^8 in the Maclaurin series for $\cos(x^2)$?

a. $\dfrac{1}{8 \cdot 7 \cdot 6 \cdot 5}$

b. $4!$

c. $\dfrac{1}{4!}$

d. $8 \cdot 7 \cdot 6 \cdot 5$

e. $\dfrac{1}{8!}$

Work Out (Points indicated. Part credit possible. Show all your work.)

14. (10 points) Estimate $\int_0^{0.1} \sin(x^2)\, dx$ to within an error of $|E| < 10^{-6}$.
Be sure to say why the error is less than 10^{-6}.
HINT: Use a Maclaurin series.

15. (20 points) Find the radius and interval of convergence of $\sum_{n=2}^{\infty} \frac{(x-3)^n}{4\ln n}$. Be sure to check the endpoints. Name or state any test you use and check the conditions.

Radius:

Interval:

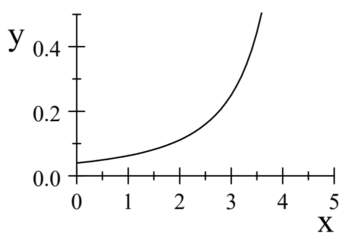
16. (10 points) The Taylor Remainder Theorem states:

If a function $f(x)$ is approximated by its k^{th} degree Taylor polynomial
$T_k f(x) = \sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!}(x-a)^n$ then the Taylor remainder $R_k f(x) = f(x) - T_k f(x)$
is $R_k f(x) = \frac{f^{(k+1)}(c)}{(k+1)!}(x-a)^{k+1}$ for some c between a and x.

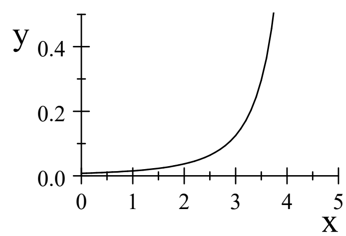
The Maclaurin series for $f(x) = \frac{1}{5-x}$ is

$$Tf(x) = \sum_{n=0}^{\infty} \frac{x^n}{5^{n+1}} = \frac{1}{5} + \frac{1}{25}x + \frac{1}{125}x^2 + \frac{1}{625}x^3 + O(x^4)$$

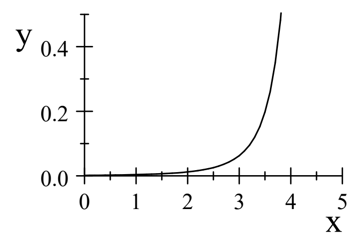
If the 2^{nd} degree Taylor polynomial, $T_2 f(x) = \frac{1}{5} + \frac{1}{25}x + \frac{1}{125}x^2$, is used to approximate $\frac{1}{5-x}$ at $x = 3$, use the Taylor Remainder Theorem to find a bound on the absolute value of the Taylor remainder, $|R_2 f(3)|$. Here are three plots:



$\frac{1}{(5-x)^2}$



$\frac{1}{(5-x)^3}$



$\frac{1}{(5-x)^4}$