

Name \_\_\_\_\_

MATH 172H

Exam 2

Spring 2021

Sections 200

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Multiple Choice and Short Answer: (Points indicated. No Part Credit)

1-11	/59	13	/15
12	/15	14	/15
		Total	/104

1. (5 pts) How many terms are there in the general partial fraction expansion of

$$\frac{6 + 7x}{(x - 2)^2(x^2 - 4)(x^2 + 4)}?$$

Note:  $\frac{A}{(x - 2)^2}$  and  $\frac{Bx + C}{x^2 + 4}$  each count as 1 term.

The number of terms is

**Answer:**  $n =$  \_\_\_\_\_

2. (5 pts) Find the coefficients in the partial fraction decomposition

$$\frac{x - 1}{x^2 - 5x + 6} = \frac{A}{x - 3} + \frac{B}{x - 2}$$

Then compute  $A - 2B$ .

**Answer:**  $A - 2B =$  \_\_\_\_\_

3. (5 pts) Given that  $\frac{32}{x^4 - 16} = \frac{1}{x - 2} - \frac{1}{x + 2} - \frac{4}{x^2 + 4}$  compute  $\int_0^1 \frac{32}{x^4 - 16} dx$ .

- a.  $-\ln 3 - \arctan \frac{1}{2}$
- b.  $-\ln 3 - 2 \arctan \frac{1}{2}$
- c.  $\ln 2 - \ln 3 - \arctan \frac{1}{2}$
- d.  $\ln 2 - \ln 3 - 2 \arctan \frac{1}{2}$
- e.  $2 \ln 2 - \ln 3 - \arctan \frac{1}{2}$
- f.  $2 \ln 2 - \ln 3 - 2 \arctan \frac{1}{2}$

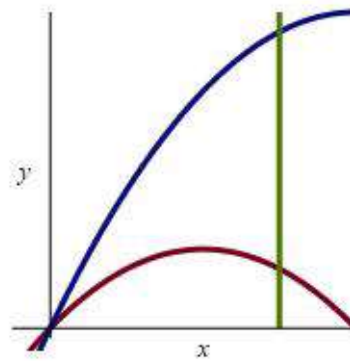
4. (5 pts) The region between  $x = 25 - y^2$  and the  $y$ -axis is rotated about the  $y$ -axis. Find the volume.

- a.  $\frac{2^4 5^4}{3} \pi$
- b.  $\frac{2^4 5^3}{3} \pi$
- c.  $\frac{2^3 5^4}{3} \pi$
- d.  $2^3 5^5 3 \pi$
- e.  $2^2 5^4 3 \pi$

5. (5 pts) The base of a solid is the region bounded by  $y = 4x - x^2$  and  $y = 8x - x^2$  and  $x = 3$ .

The slices perpendicular to the  $x$ -axis are semicircles with a diameter on the base. Find the volume.

- |            |             |
|------------|-------------|
| a. $9\pi$  | g. $72\pi$  |
| b. $12\pi$ | h. $96\pi$  |
| c. $18\pi$ | i. $150\pi$ |
| d. $24\pi$ | j. $210\pi$ |
| e. $36\pi$ | k. $270\pi$ |
| f. $48\pi$ | l. $360\pi$ |



6. (5 pts) The region bounded by  $y = 4x - x^2$  and  $y = 8x - x^2$  and  $x = 3$  (See figure above.) is rotated about the  $x$ -axis. Find the volume.

- |            |             |
|------------|-------------|
| a. $9\pi$  | g. $72\pi$  |
| b. $12\pi$ | h. $96\pi$  |
| c. $18\pi$ | i. $150\pi$ |
| d. $24\pi$ | j. $210\pi$ |
| e. $36\pi$ | k. $270\pi$ |
| f. $48\pi$ | l. $360\pi$ |

7. (5 pts) The region bounded by  $y = 4x - x^2$  and  $y = 8x - x^2$  and  $x = 3$  (See figure above.) is rotated about the  $y$ -axis. Find the volume.

- |            |             |
|------------|-------------|
| a. $9\pi$  | g. $72\pi$  |
| b. $12\pi$ | h. $96\pi$  |
| c. $18\pi$ | i. $150\pi$ |
| d. $24\pi$ | j. $210\pi$ |
| e. $36\pi$ | k. $270\pi$ |
| f. $48\pi$ | l. $360\pi$ |

8. (5 pts) Compute the improper integral  $\int_1^{\infty} xe^{-x} dx$ .

- a. 0
- b.  $\frac{1}{e}$
- c.  $\frac{2}{e}$
- d.  $\frac{4}{e}$
- e.  $\infty$

9. (5 pts) Compute the improper integral  $\int_0^1 \frac{2}{\sqrt{1-x^2}} dx$ .

- a.  $\pi$
- b.  $\frac{\pi}{2}$
- c.  $\frac{\pi}{3}$
- d.  $\frac{\pi}{4}$
- e. divergent

10. (5 pts) Compute the improper integral  $\int_0^{16} \frac{1}{(x-8)^{4/3}} dx$ .

- a. 0
- b.  $-\frac{3}{4}$
- c.  $-\frac{3}{2}$
- d. -3
- e. divergent

11. (9 pts) The rest position of a certain spring is at  $x = 0$  cm. It takes 72 ergs of work to stretch it from  $x = 4$  cm to  $x = 8$  cm.

a. Find the spring constant.

$$k = \underline{\hspace{2cm}} \frac{\text{dynes}}{\text{cm}}$$

b. How much work does it take to stretch it from  $x = 2$  cm to  $x = 6$  cm?

$$W = \underline{\hspace{2cm}} \text{ ergs}$$

c. How much force is needed to hold it at  $x = 5$  cm?

$$F = \underline{\hspace{2cm}} \text{ dynes}$$

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 pts) Find the partial fraction expansion for  $\frac{2x+9}{x^3+9x} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$ .

(3 pts Extra Credit for a complex number solution.)

$A = \underline{\hspace{2cm}}$        $B = \underline{\hspace{2cm}}$        $C = \underline{\hspace{2cm}}$

13. (15 pts) Determine if the improper integral  $\int_2^{\infty} \frac{2}{e^x+x} dx$  converges or diverges.

Do the integral exactly or use a Comparison Test.

If you do the integral exactly, be sure to state all substitutions you make and their differentials.

If you use a comparison, be sure to state the comparison integral, explain why the comparison integral converges or diverges and check the inequality.

(You will be graded for good sentences!)

Convergent       Divergent

14. (15 pts) A cone is 12 cm tall and 6 cm in radius at the top.

It is filled with salt water of density  $\delta = 1.02 \frac{\text{gm}}{\text{cm}^3}$  to a depth of 8 cm.

Find the work done to pump all the water over the top of the cone.

For numerical computations, use the approximation that

$$\delta g = 9.8 \cdot 1.02 \approx 10 \frac{\text{gm} \cdot \text{cm}}{\text{sec}^2}.$$

$W = \underline{\hspace{2cm}}$

