Name_____

1-11	/55	13	/15
12	/20	14	/15
		Total	/105

MATH 172H Exam 3 Spring 2021

Sections 200 P. Yasskin

Multiple Choice and Short Answer: (Points indicated.)

1. (5 pts) Compute $\lim_{n\to\infty}\left(\sqrt{n^2-4n+3}-\sqrt{n^2+5n-2}\right)$.

a. 0

b. -9

c. $-\frac{9}{2}$

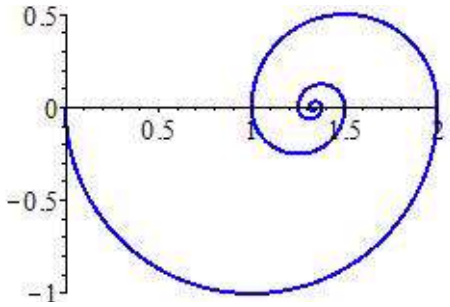
d. $\frac{9}{2}$

e. 9

2. (5 pts) Compute $L=\lim_{n\to\infty} n^{1/n}$ (Type infinity for ∞, pi for π and e for e.)

$L=$ ____

3. (5 pts) The spiral at the right is made from an infinite number of semicircles whose centers are all on the x-axis. The first semicircle has radius $r_1=1$. The radius of each subsequent semicircle is half of the radius of the previous semicircle. Find the total length of the spiral. (Type infinity for ∞, pi for π and e for e.)



$L=$ ____

4. (5 pts) Compute $\displaystyle\sum_{n=3}^{\infty}\left(\frac{\sqrt{n}}{\sqrt{n+1}}-\frac{\sqrt{n+1}}{\sqrt{n+2}}\right)$

a. $\dfrac{\sqrt{3}}{2}$

b. $\dfrac{2-\sqrt{3}}{2}$

c. 0

d. $\dfrac{\sqrt{3}-2}{2}$

e. $\dfrac{-\sqrt{3}}{2}$

5. (5 pts) Which of the following are correct about the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+\sqrt{n}}$?

Answer all that are correct.

Scoring: Grade $=\dfrac{\#\text{ answered correctly}}{\#\text{ correct answers}}\cdot 5-\#\text{ answered incorrectly}$

a. diverges by the n^{th} Term Divergence Test

b. diverges by the Simple Comparison Test comparing to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

c. diverges by the Limit Comparison Test comparing to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

d. converges because it is a p-series

e. converges by the Simple Comparison Test comparing to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$

f. converges by the Limit Comparison Test comparing to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$

g. converges by the Ratio Test

6. (5 pts) Find a power series about $x = 0$ for $f(x) = \dfrac{4x^3}{1-x^2}$.

a. $\displaystyle\sum_{n=0}^{\infty}(4x^3)^{2n}$

b. $\displaystyle\sum_{n=0}^{\infty}8nx^{2n+3}$

c. $\displaystyle\sum_{n=0}^{\infty}4x^{2n+3}$

d. $\displaystyle\sum_{n=0}^{\infty}4x^{2(n+3)}$

e. $\displaystyle\sum_{n=0}^{\infty}4nx^{2n+3}$

f. $\displaystyle\sum_{n=0}^{\infty}4nx^{2(n+3)}$

7. (5 pts) Find a power series about $x = 0$ for $f(x) = \dfrac{2x}{(1-x^2)^2}$.

a. $\displaystyle\sum_{n=0}^{\infty}2nx^{2n-1}$

b. $\displaystyle\sum_{n=0}^{\infty}2x^{2n-1}$

c. $\displaystyle\sum_{n=0}^{\infty}2nx^{2n+1}$

d. $\displaystyle\sum_{n=0}^{\infty}2x^{2n+1}$

e. $\displaystyle\sum_{n=0}^{\infty}4n^3x^{2n-1}$

f. $\displaystyle\sum_{n=0}^{\infty}4n^3x^{2n+1}$

8. (5 pts) Find the Taylor series for $f(x) = \dfrac{1}{x}$ about $x = 2$.

a. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^n}x^n$

b. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^n}(x-2)^n$

c. $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{2^n}x^n$

d. $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{2^n}(x-2)^n$

e. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}x^n$

f. $\displaystyle\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}(x-2)^n$

g. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}x^n$

h. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}(x-2)^n$

i. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n n!}{2^n}x^n$

j. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n n!}{2^n}(x-2)^n$

k. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}x^n$

l. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}(x-2)^n$

9. (5 pts) Use the 3^{rd} degree Taylor polynomial for $\sin(x)$ centered at $x = 0$ to approximate $\sin(0.3)$.

a. $.3$

b. $.309$

c. $.291$

d. $.3045$

e. $.2955$

10. (5 pts) Compute $S = \sum_{n=0}^{\infty} \frac{1}{2^n n!}$

a. $\sin(2)$

b. $\sin\left(\frac{1}{2}\right)$

c. $\frac{\sin(1)}{2}$

d. e^2

e. $\sqrt{e}$

f. $\frac{e}{2}$

g. $\cos(2)$

h. $\cos\left(\frac{1}{2}\right)$

i. $\frac{\cos(1)}{2}$

j. -1

k. 2

l. ∞

11. (5 pts) Compute $L = \lim_{x \to \infty} \frac{1 - \cos(2x)}{x^2}$

$L =$ ____

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (20 pts) Work Out Problem

For each power series, find the radius and interval of convergence.
Give complete explanations. (Type infinity for ∞.)

a. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)}(x-3)^n$

$R =$ ____ $I =$ _____

b. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n(n+1)!}(x-3)^n$

$R =$ ____ $I =$ _____

13. (15 pts) Work Out Problem

Consider the sequence given by the recursion relation $a_{n+1} = 2\sqrt{a_n}$ starting from $a_1 = 1$.
Does the sequence have a limit? If so, find the limit. If not, enter divergent.
Be sure to use sentences, name the theorem you use and prove all statements.

$\lim_{n\to\infty} a_n =$ ____

14. (15 pts) Work Out Problem

Give a complete explantion as to why the series $\sum_{n=2}^{\infty} \frac{(-1)^n(n+1)}{n^2+\sqrt{n}}$ is absolutely convergent, conditionally convergent or divergent.

a. absolutely convergent

b. conditionally convergent

c. divergent