Name________________________________

MATH 172 Honors Final Exam Spring 2022

Sections 200 P. Yasskin

1-10	/50	14	/10
11	/10	15	/10
12	/5	16	/15
13	/5	Total	/105

Multiple Choice: (5 points each. Circle your answers.)
Show your work in case I want to look at it.

1. (5 points) Find the average value of the function $f(x) = x\sqrt{x^2+1}$ on the interval $\left[0, \sqrt{8}\,\right]$.

a. $\dfrac{13}{3\sqrt{8}}$ d. $\dfrac{26}{3\sqrt{8}}$ g. $\dfrac{13}{\sqrt{8}}$ j. $\dfrac{52}{3\sqrt{8}}$

b. $\dfrac{13}{3}$ e. $\dfrac{26}{3}$ h. 13 k. $\dfrac{52}{3}$

c. $\dfrac{13\sqrt{8}}{3}$ f. $\dfrac{26\sqrt{8}}{3}$ i. $13\sqrt{8}$ l. $\dfrac{52\sqrt{8}}{3}$

2. (5 points) Find the arclength of the curve $\vec{r}(t) = \langle t\cos t - \sin t, t\sin t + \cos t\rangle$ between $t = 0$ and $t = 1$.

a. $\frac{1}{2}\ln\left(\sqrt{2}+1\right) - \frac{1}{2}\sqrt{2}$ d. $\ln\left(\sqrt{2}+1\right) - \sqrt{2}$ g. $\frac{1}{8}$ j. $\dfrac{\sqrt{2}}{8}$

b. $\frac{1}{2}\ln\left(\sqrt{2}+1\right) + \frac{1}{2}\sqrt{2}$ e. $\ln\left(\sqrt{2}+1\right) + \sqrt{2}$ h. $\frac{1}{4}$ k. $\dfrac{\sqrt{2}}{4}$

c. $\frac{1}{2}\ln\left(\sqrt{2}+1\right)$ f. $\ln\left(\sqrt{2}+1\right)$ i. $\frac{1}{2}$ l. $\dfrac{\sqrt{2}}{2}$

3. (5 points) A 60 lb force stretches a spring 3 ft from its rest position.
How much work is done to stretch the spring from its rest position to 4 ft from its rest position?

a. 12 ft-lb
b. 20 ft-lb
c. 24 ft-lb
d. 60 ft-lb
e. 80 ft-lb
f. 90 ft-lb
g. 160 ft-lb
h. 180 ft-lb
i. 240 ft-lb
j. 320 ft-lb
k. 720 ft-lb
l. 1440 ft-lb

4. (5 points) The base of a solid is the region between $y = 4 - x^2$ and the x-axis.
The cross sections perpendicular to the x-axis are squares. Find its volume.

a. $\frac{32}{35}$
b. $\frac{128}{35}$
c. $\frac{256}{35}$
d. $\frac{512}{35}$
e. $\frac{32}{15}$
f. $\frac{128}{15}$
g. $\frac{256}{15}$
h. $\frac{512}{15}$
i. $\frac{32}{5}$
j. $\frac{128}{5}$
k. $\frac{256}{5}$
l. $\frac{512}{5}$
m. $\frac{32}{3}$
n. $\frac{128}{3}$
o. $\frac{256}{3}$
p. $\frac{512}{3}$

5. (5 points) Compute $\int_0^2 2x^3 e^{x^2}\,dx$.

a. $\frac{3}{2}e^4 - 1$ d. $3e^4 - 1$ g. $4e^4 - 1$ j. $5e^4 - 1$

b. $\frac{3}{2}e^4$ e. $3e^4$ h. $4e^4$ k. $5e^4$

c. $\frac{3}{2}e^4 + 1$ f. $3e^4 + 1$ i. $4e^4 + 1$ l. $5e^4 + 1$

6. (5 points) After making the appropriate trig substitution, $\int_0^1 \frac{1}{(x^2-4)^8}\,dx$ becomes

a. $\frac{1}{2^{15}}\int_{x=0}^{1} \sec^9\theta\,d\theta$ e. $\frac{1}{2^{15}}\int_{x=0}^{1} \sec^{15}\theta\,d\theta$ i. $\frac{1}{2^{15}}\int_{x=0}^{1} \sec^{15}\theta\tan\theta\,d\theta$

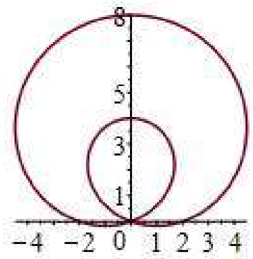
b. $\frac{1}{2^{15}}\int_{x=0}^{1} \frac{1}{\sec^{14}\theta}\,d\theta$ f. $\frac{1}{2^{15}}\int_{x=0}^{1} \frac{\sec\theta}{\tan^{15}\theta}\,d\theta$ j. $\frac{1}{2^{15}}\int_{x=0}^{1} \frac{\tan\theta}{\sec^{14}\theta}\,d\theta$

c. $\frac{1}{2^{16}}\int_{x=0}^{1} \sec^{10}\theta\,d\theta$ g. $\frac{1}{2^{16}}\int_{x=0}^{1} \sec^{16}\theta\,d\theta$ k. $\frac{1}{2^{16}}\int_{x=0}^{1} \sec^{16}\theta\tan\theta\,d\theta$

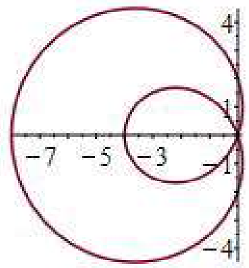
d. $\frac{1}{2^{16}}\int_{x=0}^{1} \frac{1}{\sec^{16}\theta}\,d\theta$ h. $\frac{1}{2^{16}}\int_{x=0}^{1} \frac{1}{\tan^{16}\theta}\,d\theta$ l. $\frac{1}{2^{16}}\int_{x=0}^{1} \frac{\tan\theta}{\sec^{16}\theta}\,d\theta$

7. (5 points) Which of the following is the graph of the polar equation $r = 2 - 6\sin\theta$?

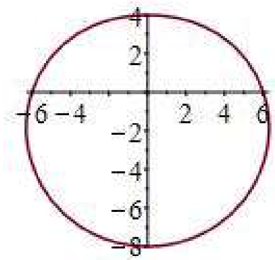
a.



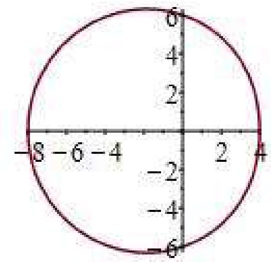
c.



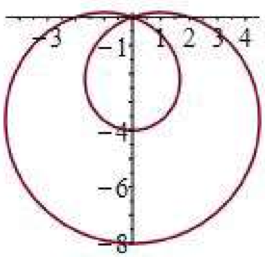
e.



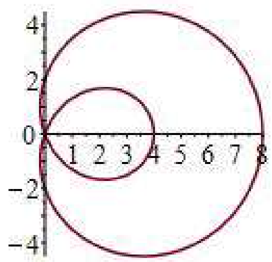
g.



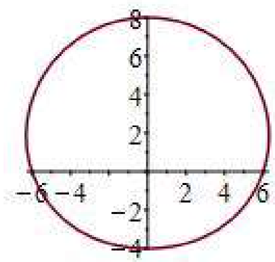
b.



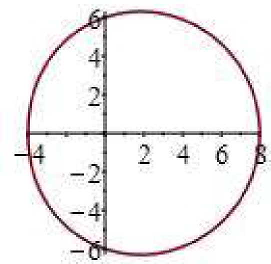
d.



f.



h.



8. (5 points) Solve the initial value problem $\frac{dy}{dx} = \frac{y^{2/3}}{x^{2/3}}$ with $y(0) = 1$. Then find $y(1)$.

a. $y(1) = 0$

b. $y(1) = 1$

c. $y(1) = e$

d. $y(1) = 2$

e. $y(1) = 4$

f. $y(1) = 8$

g. $y(1) = 2^{1/3}$

h. $y(1) = 4^{1/3}$

i. $y(1) = e^{1/3}$

j. $y(1) = 2^{1/3} + 1$

k. $y(1) = 4^{1/3} + 1$

l. $y(1) = e^{1/3} + 1$

9. (5 points) Find the sum of the series $\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!}$.

a. $-\pi$	d. -1	g. 2
b. $-\frac{\pi}{2}$	e. 0	h. $\frac{\pi}{2}$
c. -2	f. 1	i. π

10. (5 points) If $f(x) = x^2 e^x$, find $f^{(11)}(0)$, the 11^{th} derivative at 0.

a. $9!$	d. $\frac{1}{9!}$	g. 72	j. $\frac{1}{72}$
b. $10!$	e. $\frac{1}{10!}$	h. 90	k. $\frac{1}{90}$
c. $11!$	f. $\frac{1}{11!}$	i. 110	l. $\frac{1}{110}$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (10 points) Find the partial fraction expansion for $\frac{3x-2}{x^3+x}$ and compute $\int \frac{3x-2}{x^3+x}\,dx$.

12. (5 points) Solve the initial value problem $\frac{dy}{dx} + \frac{y}{x} + x^3 = 0$ with $y(5) = 0$. Then find $y(-5)$.

13. (5 points) The series $\sum_{n=0}^{\infty}(n+1)x^n$ converges to the function $\frac{1}{(1-x)^2}$ on $(-1,1)$.

What function does the series $\sum_{n=0}^{\infty}(n+1)nx^{n-1}$ converges to on $(-1,1)$?

14. (10 points) Compute $\lim_{x\to 0} \frac{\sin x - x\cos x}{x^3}$.

15. (10 points) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{3^n(2n+1)^3}$.

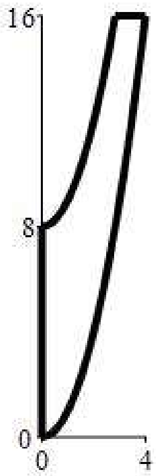
Be sure to explain any test you use.

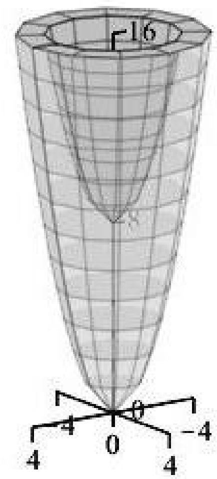
16. (15 points) The region in the first quadrant bounded by

$x = \sqrt{y}$, $x = \sqrt{y-8}$, $x = 0$ and $y = 16$

is revolved about the y-axis to form a bowl.

HINT: You will need to split the integrals at $y = 8$.





a.(5 pts) Find the voume swept out.

b. (5 pts) If the bowl is fillled with water, find the work done to pump the water out the top of the bowl. Give the answer as a multiple of $g\delta$ where g is the acceleration of gravity and δ is the density of water. Once you plug in numbers, you do not need to simplify.

(16. continued)

(16. continued)

c. (5 pts Extra Credit)   Find the height of the centroid of the bowl (without water).
(The volume was found in part a.)