Name________________________

1-9	/54	11	/20
10	/ 5	12	/25
		Total	/104

MATH 221 Exam 2 Version A Fall 2019

Section 504 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Find the equation of the plane tangent to $z = x^2y^2 + xy^2$ at the point $(x,y) = (1,2)$.
Its z-intercept is:

a. $c = 20$

b. $c = 8$

c. $c = -8$

d. $c = -16$

e. $c = -20$

2. Use differentials to estimate the volume of metal needed to make a cylindrical tin can with lids if the radius is $r = 5$ cm and the height is $h = 6$ cm and the metal has thickness $.02$ cm?

a. 200π

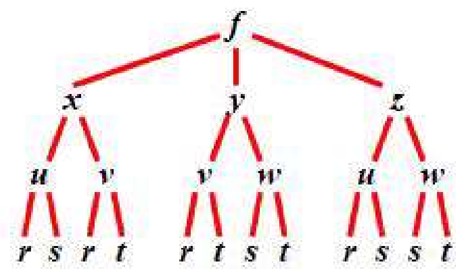
b. 4π

c. 105π

d. 2.2π

e. 2.6π

3. At the right is a tree diagram showing f as a function of x, y and z which are functions of u, v and w which are functions of r, s and t as indicated. Below are values of a bunch partial derivatives. Use this information to compute $\frac{\partial f}{\partial s}$.



$$\frac{\partial f}{\partial x} = 2 \qquad \frac{\partial f}{\partial y} = 3 \qquad \frac{\partial f}{\partial z} = 4$$

$$\frac{\partial x}{\partial u} = 5 \qquad \frac{\partial x}{\partial v} = 6 \qquad \frac{\partial y}{\partial v} = 7 \qquad \frac{\partial y}{\partial w} = 8 \qquad \frac{\partial z}{\partial u} = 9 \qquad \frac{\partial z}{\partial w} = 10$$

$$\frac{\partial u}{\partial r} = 6 \qquad \frac{\partial u}{\partial s} = 5 \qquad \frac{\partial v}{\partial r} = 4 \qquad \frac{\partial v}{\partial t} = 3 \qquad \frac{\partial w}{\partial s} = 2 \qquad \frac{\partial w}{\partial t} = 1$$

a. 163

b. 212

c. 358

d. 396

e. 408

4. The point $(x,y) = (1,2)$ is a critical point of the function $f = 8x^3 + y^3 - 12xy$.
Use the 2^{nd} Derivative Test to classify it as:

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. The 2^{nd} Derivative Test FAILS.

5. If x, y and z are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial x}$ at the point $(x,y,z) = \left(3, \frac{\pi}{3}, \sqrt{3}\right)$.

a. $\frac{1}{\sqrt{3}}$

b. $\frac{-1}{\sqrt{3}}$

c. $\sqrt{3}$

d. $-\sqrt{3}$

e. $\frac{1}{3}$

6. If x, y and z are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial t}$ at the instant when:

$$(x,y,z) = \left(3, \frac{\pi}{3}, \sqrt{3}\right) \qquad \frac{dx}{dt} = \sqrt{3} \qquad \frac{dy}{dt} = 1$$

a. 1

b. 2

c. 3

d. $\sqrt{3}$

e. $\frac{1}{\sqrt{3}}$

7. Find the tangent plane to the graph of the equation $xy - zy = 4$ at the point $(x,y,z) = (3,2,1)$. Its z-intercept is:

a. $c = -8$

b. $c = -4$

c. $c = 0$

d. $c = 4$

e. $c = 8$

8. Queen Lena is flying the Centurion Eagle through a deadly Sythion field whose density is $S = xyz \dfrac{\text{Sythions}}{\text{microlightyear}^3}$. The top speed of the Centurion Eagle is $14 \dfrac{\text{microlightyears}}{\text{lightyear}}$. If Lena is located at the point $(x, y, z) = (1, 2, 3)$, what should her velocity be to **decrease** the Sythion density as fast as possible?

a. $\langle 6, 3, 2 \rangle$

b. $\langle -84, -42, -28 \rangle$

c. $\langle -6, -3, -2 \rangle$

d. $\langle 12, 6, 4 \rangle$

e. $\langle -12, -6, -4 \rangle$

9. Consider the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^3 + y^6}$. Which of the following directions of approach gives a different value of the limit?

a. Non-vertical line: $y = mx$ and $x \to 0$

b. The y-axis: $x = 0$ and $y \to 0$

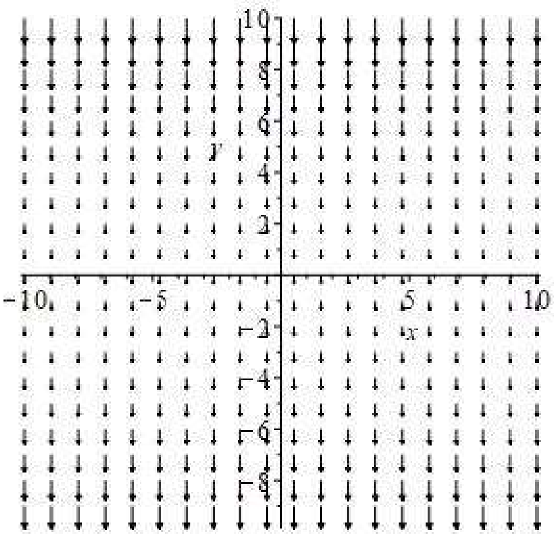
c. The parabola: $x = y^2$ and $y \to 0$

d. The parabola: $y = x^2$ and $x \to 0$

e. None of these. They all give the same limit.

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (5 points) Here is the plot of a vector field $\vec{F}$ in $\mathbb{R}^2$.
Shade in the region where $\vec{\nabla} \cdot \vec{F} > 0$. Explain why.



11. (20 points) Find a scalar potential, f, for $\vec{F} = \langle yz^2 - 2xz, xz^2 - 3y^2z, 2xyz - x^2 - y^3 + 2z \rangle$ or show one does not exist. Explain all steps neatly and clearly.

12. (25 points) Find the largest and smallest values of the function $f(x,y,z) = xyz$ on the ellipsoid $x^2 + 4y^2 + 16z^2 = 48$.