Name_____

MATH 221 Exam 2 Version A Fall 2019

Section 505 P. Yasskin

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|---|---|---|---|
| 1-9 | /54 | 11 | /20 |
| 10 | / 5 | 12 | /25 |
| | | Total | /104 |

Multiple Choice: (6 points each. No part credit.)

**1**. Find the equation of the plane tangent to $z = x^2y^2 + xy^2$ at the point $(x,y) = (1,2)$.
Its $z$-intercept is:

**a**. $c = 20$

**b**. $c = 8$

**c**. $c = -8$

**d**. $c = -16$

**e**. $c = -20$

**2**. Use differentials to estimate the volume of metal needed to make a cylindrical tin can with lids if the radius is $r = 5$ cm and the height is $h = 6$ cm and the metal has thickness $.02$ cm?

**a**. $200\pi$

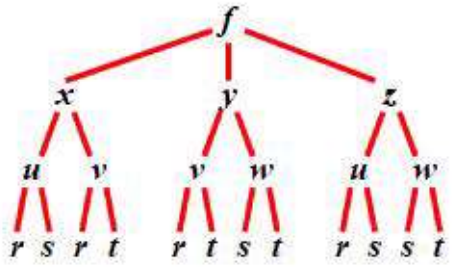
**b**. $4\pi$

**c**. $105\pi$

**d**. $2.2\pi$

**e**. $2.6\pi$

**3**. At the right is a tree diagram showing $f$ as a function of $x$, $y$ and $z$ which are functions of $u$, $v$ and $w$ which are functions of $r$, $s$ and $t$ as indicated. Below are values of a bunch partial derivatives. Use this information to compute $\frac{\partial f}{\partial s}$.




$$\frac{\partial f}{\partial x} = 2 \quad \frac{\partial f}{\partial y} = 3 \quad \frac{\partial f}{\partial z} = 4$$

$$\frac{\partial x}{\partial u} = 5 \quad \frac{\partial x}{\partial v} = 6 \quad \frac{\partial y}{\partial v} = 7 \quad \frac{\partial y}{\partial w} = 8 \quad \frac{\partial z}{\partial u} = 9 \quad \frac{\partial z}{\partial w} = 10$$

$$\frac{\partial u}{\partial r} = 6 \quad \frac{\partial u}{\partial s} = 5 \quad \frac{\partial v}{\partial r} = 4 \quad \frac{\partial v}{\partial t} = 3 \quad \frac{\partial w}{\partial s} = 2 \quad \frac{\partial w}{\partial t} = 1$$

**a**. 163

**b**. 212

**c**. 358

**d**. 396

**e**. 408

**4**. The point $(x,y) = (1,2)$ is a critical point of the function $f = 8x^3 + y^3 - 12xy$. Use the $2^{\text{nd}}$ Derivative Test to classify it as:

**a**. Local Minimum

**b**. Local Maximum

**c**. Inflection Point

**d**. Saddle Point

**e**. The $2^{\text{nd}}$ Derivative Test FAILS.

**5**. If $x,\ y$ and $z$ are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial x}$ at the point $(x,y,z) = \left(3, \frac{\pi}{3}, \sqrt{3}\right)$.

**a**. $\frac{1}{\sqrt{3}}$

**b**. $\frac{-1}{\sqrt{3}}$

**c**. $\sqrt{3}$

**d**. $-\sqrt{3}$

**e**. $\frac{1}{3}$

**6**. If $x,\ y$ and $z$ are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial t}$ at the instant when:

$$(x,y,z) = \left(3, \frac{\pi}{3}, \sqrt{3}\right) \qquad \frac{dx}{dt} = \sqrt{3} \qquad \frac{dy}{dt} = 1$$

**a**. $1$

**b**. $2$

**c**. $3$

**d**. $\sqrt{3}$

**e**. $\frac{1}{\sqrt{3}}$

**7**. Find the tangent plane to the graph of the equation $xy - zy = 4$ at the point $(x,y,z) = (3,2,1)$. Its $z$-intercept is:

**a**. $c = -8$

**b**. $c = -4$

**c**. $c = 0$

**d**. $c = 4$

**e**. $c = 8$

**8**. Queen Lena is flying the Centurion Eagle through a deadly Sythion field whose density is $S = xyz \ \frac{\text{Sythions}}{\text{microlightyear}^3}$. The top speed of the Centurion Eagle is $14 \ \frac{\text{microlightyears}}{\text{lightyear}}$. If Lena is located at the point $(x,y,z) = (1,2,3)$, what should her velocity be to **decrease** the Sythion density as fast as possible?

**a**. $\langle 6,3,2 \rangle$

**b**. $\langle -84,-42,-28 \rangle$

**c**. $\langle -6,-3,-2 \rangle$

**d**. $\langle 12,6,4 \rangle$

**e**. $\langle -12,-6,-4 \rangle$

**9**. Consider the limit: $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^3+y^6}$. Which of the following directions of approach gives a different value of the limit?

**a**. Non-vertical line: $y = mx$ and $x \to 0$

**b**. The $y$-axis: $x = 0$ and $y \to 0$

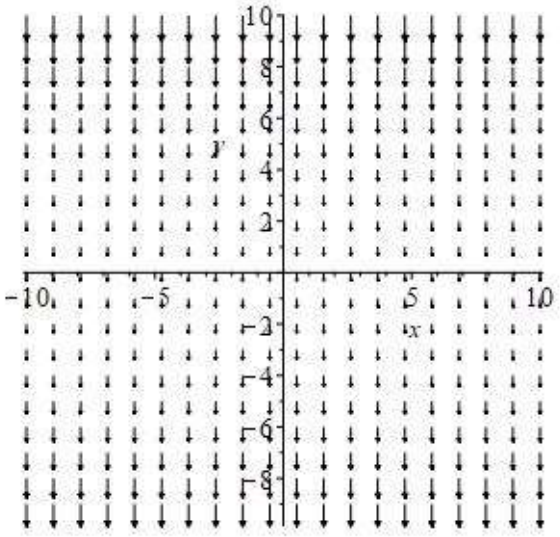
**c**. The parabola: $x = y^2$ and $y \to 0$

**d**. The parabola: $y = x^2$ and $x \to 0$

**e**. None of these. They all give the same limit.

Work Out: (Points indicated. Part credit possible. Show all work.)

**10**. (5 points) Here is the plot of a vector field $\vec{F}$ in $\mathbb{R}^2$.
Shade in the region where $\vec{\nabla} \cdot \vec{F} > 0$. Explain why.




**11**. (20 points) Find a scalar potential, $f$, for $\vec{F} = \langle yz^2 - 2xz, xz^2 - 3y^2z, 2xyz - x^2 - y^3 + 2z \rangle$ or show one does not exist. Explain all steps neatly and clearly.

**12**. (25 points) Find the largest and smallest values of the function $f(x,y,z) = xyz$ on the ellipsoid $x^2 + 4y^2 + 16z^2 = 48$.