Name	UIN					
			1-8	/48	10	/20+5
MATH 221	Exam 2	Fall 2021				
Sections 504/505		P. Yasskin	9	/15	11	/15
Multiple Choice	e: (6 points each. No	part credit.)			Total	/108

1. Find the equation of the plane tangent to $z = x^2y^4 - \frac{x}{y}$ at (x,y) = (2,1). Then find the *z*-intercept.

z = _____

2. Find the plane tangent to the hyperboloid $4x^2 + 9y^2 - 36z^2 = 36$ at the point (x, y, z) = (3, 2, 1). Write the plane in the form where the right side is 1.

$$\underline{\qquad \qquad } x + \underline{\qquad \qquad } y + \underline{\qquad \qquad } z = 1$$

- **3**. A weather balloon takes measurements at the point (x,y,z) = (5,8,3) km. It finds the barometric pressure is P = 1.05 atm and its gradient is $\vec{\nabla}P = \langle .02, -.03, .04 \rangle$. Estimate the pressure at (x,y,z) = (4.7,7.8,3.2) km. $P(4.7,7.8,3.2) \approx$
- 4. The Ideal Gas Law says the Pressure, *P*, Density, δ , and Temperature, *T*, are related by $P = k\delta T$. A particular sample of ideal gas has k = 2. At a certain point the pressure, density and temperature are P = 4 $\delta = .01$ T = 200

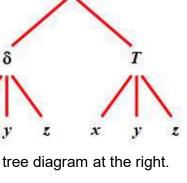
The gradients of the density and temperature are

 $\vec{\nabla}\delta = \langle .001, .002, .003 \rangle$ $\vec{\nabla}T = \langle 3, 2, 1 \rangle$

Find the gradient of the pressure.

Hint: Compute each component separately using the chain rule and the tree diagram at the right.

a. $\vec{\nabla}P = \langle .46, .48, 1.22 \rangle$ **e**. $\vec{\nabla}P = \langle .46, .48, 2.11 \rangle$ **b**. $\vec{\nabla}P = \langle .64, .48, 1.22 \rangle$ **f**. $\vec{\nabla}P = \langle .64, .48, 2.11 \rangle$ **c**. $\vec{\nabla}P = \langle .46, .84, 1.22 \rangle$ **g**. $\vec{\nabla}P = \langle .46, .84, 2.11 \rangle$ **d**. $\vec{\nabla}P = \langle .64, .84, 1.22 \rangle$ **h**. $\vec{\nabla}P = \langle .64, .84, 2.11 \rangle$

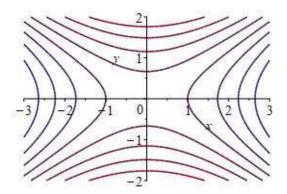


Ideal Gas Law Tree Diagram

P

- **5**. The point (x,y) = (0,2) is a critical point of the function $f(x,y) = y^4 32y + 8x^2y$. Use the Second Derivative Test to classify the point or say the test fails.
 - a. Local Minimum
 - b. Local Maximum
 - c. Inflection Point
 - d. Saddle Point
 - e. Test FAILS

6. The contour plot of a function f is shown. Which of the following is the plot of its gradient, $\vec{\nabla}f$?



С.	
----	--

1	1	1	1	1	1	1	1	I.	2-	1	1	1	1	1	1	1	X	1	1
1	1	1	1	1	1	1	1	1	Т	1	-13	1	1	1	1	1	X	1	1
1	1	1	1	1	1	1	1	I.	1.	1	10	1	1	A.	1	1	1	1	1
1	1	1	1	1	1	1	1	£	1	4	- 1	1	- 1	N.	1	1	1	X	1
1	1	1	1	1	1	1	1.	1	1	Э£.	- 10	. 3	X	N	1	1	N.	1	8
1	1	1	1	1	1	1	1	1	1-		×.	. 5		×.		5	N .	~	~
1	1	1	1	1	1	1	1	x			×.			5	~	-	*	~	~
-	1		1		1			10	184	10	- 62	×.			. •		5	-	-
-	*			-				2	10					*		-	-	-	-
-	-	-	-	-	14	÷.	. ÷.	\mathcal{X}	×4.,	$\langle \hat{u} \rangle$	- 21			- 40	-	-	4	+	-
÷.,	+	÷.,	-	-	6	1.0	. t.	1	÷.	10	- 60	•	10	10	. e.	2	-	-	-
3	*		-	2-	1.	-	1.	18	0	4	- 60		1-	۰.	*	2	-	-	3
-					1.21			- 29	100.00				11.5	- 227					
	~	~			1.20			- 20	1000	100	- 50		1	- C.	1	1	100	-	
~	-	-	-				2	3		4	1	i.	1	2		1	2	2	2
11	111	111	111		:			1		1		;		2	1	1	1	1	1
111	1111	1111	1111					1	1-	1 1		11		211	"."	111	1111	1111	111
1111	11111	11111	11111						1-1-1	1 1 1				2111		11111	11111	11111	1111
11111	111111	111111	111111	111111						1 1 1 1				121111		11111	111111	111111	11111
~~~~~ hT	1111111	1111111	1111111	1111111						1 1 1 1 1 1				11111	~~~~~~~~~~~~	1111111	*****	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	11/1/11

~	~	~	1	-	-	-	-	-	2	-	-	-	-	-	-	1	1	1	
~	~	~	~	~	-	-	-	-	-	-	-	-	-	-	1	1	1	1	۰.
~	~	~	~	~	-	~	-	-	-	-	-	-	-	-	2	1	1	1	۰,
1	1	~	*	~	~	*	-	-	-	-	-	-	-	-	1	1	1	1	1
1	1	~		5	~	-	-	-	-	-		-	-	1	1	1	1	1	2
1	1	1					Y	-	1-	-	-	~		1	1	1	1	1	15
1	×	1					80		1.0	-	-			1	1	1	1	1	12
1	1	. 1											1	1	1	1	1	1	
1	1	٠.				1.4		+	÷.	$\langle \bullet \rangle$				1		1	1	1	
- N.	. 1	1	З¥.,	. 1	. Ē	4.		- 42	94.),		- 80	÷.,				10	1	1	
- T.	. 1	1	1			1.	. •		~ A	10	- 81	•	1			5	3	1	12
	51	-t	7	21		-	1.	- 41	0		- 53		1.	۸.		2	×.	1	3
1	1	1	1	1	1		*	45									×.	. 8	
1	1	1	1	1	1		4	-	-					2	6 .		1	1	
1	1	1	1	1	1		*	-	+	-	-	-		5	~	-	N	N	2
1	1	1	1	1	1	-	-	-	-	-	-	-	-	~	~	~	1	1	9
1	1	1	1	1	1	-	-	-	-	-	-	-	-	~	~	~	~	~	1
1	1	1	1	1	-	-	-	-	-	-	-	-	-	~	~	~	~	~	6
1	1	1	1	1	-	-	-	-	-	-	-	-	-	-	~	~	~	~	6
		1	1112		100	0152	11.1	100		200					225	20	2.2		1

a.

b.

2	~	~					V	1	1-	1	1		1	1	1	1	1	1	.,
~	~	~		~			5		5	4	1			2		1	1	1	-
-	-	-	~		-		4	1			1			1		1	2	1	-
シンシン 日間 インノノノノノ	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	-	-	1	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	14	V		0	10	*********		·····	·····		· · · · · · · · · · · · · · · · · · ·	-	~~~~	1111 Han 111111111
5	-	-	14	-	-	22.	14	-45	G2.,	141	- 22		12	-	-	-	-	-	-
1		-	1	100	-			-	- 4	1.2	1		14			5	-	-	4
-3	-	-	-	2-		-	1-	10	0		- 60		1.			2	-	-	3
-	-	1	1		1		1	A.			- 63		~			~	-	-	5
1	1	1	1	1	1			1						2	1	~	~	~	-
1	1	1	1	1	1	1	1	- 6	1.	1				\$	\$			~	-
1	1	1	1	1	1	1	1	1	1	4		1	N	N.	1	- N	1	1	1
1	1	1	1	1	1	1	1	1	1	4	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	N	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	N	1	1	1	N	1
1	1	1	1	1	1	1	1	F	2-	T.	1	1	1	1	1	1	1	1	1
									_										
1	1	1	1	1	-	-	4		2	-	-	-	-	-	1	~	~	1	5
1	1	1	1	1	-	-	-	-	-	-	-	-	-	-	-	~	~	~	1
1	1	1	1	1	1	-	-	-	-		-	-	-	-	~	~	~	1	1
1	1	1	1	1	1	-	-	-	-	-	-	-	-	~	5	-	~	1	1
1	1	1	1	1	1	-	*	1	-	-	-	-	*	~	~	~	1	1	1
1	1	1	1	1		1	y	-	F	-	-	-	-	5		~	×.	1	1
1	1	1	1		1	1	*	•	-	-			*	- 60	*		N.	1	1
1	1	1	1	1	1	1	*	-				1			. 8	1	- N	٠.	1
1	1	1				18	*	- 50			- 83					. *		1	- 1
1	- 11-	1	1	<u>, r</u>	1	1	×.	- 81		, G.C.	- 21	(A.)	18	- A.;			1	1	1
1,			- 18 A	1	11	185	·*	3	ò.	100	1	•	45		1	2	1	1	4
	1	0.0	1.5%	<b>`</b>						1244	- 20	1.4	1.0	1.1	1.00		1	1	1
70	1	4	÷,	2.	1	1	1.	- 53	6		- 53	1.50	- A.	- 24	0.50	-48	- 52		-
1	1	1	7.	2,	-		1.	1	•		1		-	1	1	4	ì	1	ĩ
1			+ • •	2.		+	1.	1.5.1	9		1.1			3	ć,	4	1	1	1
2			+	2:		+	1	1.1.5	• •	1.1.1	111			121	· · · ·	4		111	111
2			+	2::::		+ + + + + + + + + + + + + + + + + + + +		1111	1	1111	1111		1111	1111		4			1111
2			++++++1	2		++++++	1	11/11/1	1	11111	11111		11111	11111		4			
2			Freitt	2		Fille	1	111/11/14	1111	CC1111	111111		111111	11111		4		111111	
ノノノノノノノノー+やく、、、、、、	///////////////////////////////////////		ノノノノノノノーー ディッシンノノノノ	2		FULLIN	1111111	1111/11/11	1111				11111111-1-1-1-1-1-1-1111	11111		111111111111111111111111111111111111111	111111111111111111111111111111111111111		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

2

7. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is  $\delta = yz + xz + xy$ . If Ham's current position is P = (1,1,3), find the rate of change of the density in the direction toward the point Q = (2,3,1).

 $\nabla_{\hat{w}}\delta =$  _____

8. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is  $\delta = yz + xz + xy$ . If Ham's current position is P = (1, 1, 3), in what unit vector direction should he travel to increase the cloaking sparkles as fast as possible?

 $\hat{u} = \langle __, __, __ \rangle$ 

Work Out: (Points indicated. Part credit possible. Show all work.)

**9**. (15 points) Consider the limit  $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ . Determine the value of the limit or show the limit does not exist. **10**. (20 points + 5 pts extra credit) Find the largest value of f = xyz on the ellipsoid  $\frac{x^2}{16} + \frac{y^2}{4} + z^2 = 3$ .

NOTE: Solve by either Eliminating a Variable or by Lagrange Multipliers. Extra Credit for solving by both methods. Draw a line across your paper to clearly separate the two solutions.

HINT: When Eliminating a Variable, maximize  $F = f^2 = x^2 y^2 z^2$ .

**11**. (15 points) For each vector field, calculate its divergence and curl. Say if it has a scalar potential or a vector potential. Find the scalar potential if it exists. Do NOT find the vector potential.

**a**. 
$$\vec{F} = \langle -xy^2, x^2y, y^2z - x^2z \rangle$$
  
 $\vec{\nabla} \cdot \vec{F} =$   
 $\vec{\nabla} \times \vec{F} =$ 

Has a scalar potential?YesNoHas a vector potential?YesNo

Find a scalar potential:

*f* = _____

**b**. 
$$\vec{G} = \langle 2xz, 2yz, x^2 + y^2 + 2z \rangle$$
  
 $\vec{\nabla} \cdot \vec{G} =$   
 $\vec{\nabla} \times \vec{G} =$ 

Has a scalar potential:YesNoHas a vector potential:YesNoFind a scalar potential:

*g* = _____