Name________________________UIN________________

MATH 221 Exam 2 Fall 2021

Sections 504/505 P. Yasskin

1-8	/48	10	/20+5
9	/15	11	/15
		Total	/108

Multiple Choice: (6 points each. No part credit.)

1. Find the equation of the plane tangent to $z = x^2y^4 - \frac{x}{y}$ at $(x,y) = (2,1)$.
Then find the z-intercept.

$z =$ __________

2. Find the plane tangent to the hyperboloid $4x^2 + 9y^2 - 36z^2 = 36$ at the point $(x,y,z) = (3,2,1)$.
Write the plane in the form where the right side is 1.

______ $x +$ ______ $y +$ ______ $z = 1$

3. A weather balloon takes measurements at the point $(x,y,z) = (5,8,3)$ km.
It finds the barometric pressure is $P = 1.05$ atm and its gradient is $\vec{\nabla}P = \langle .02, -.03, .04 \rangle$.
Estimate the pressure at $(x,y,z) = (4.7, 7.8, 3.2)$ km.

$P(4.7, 7.8, 3.2) \approx$ __________

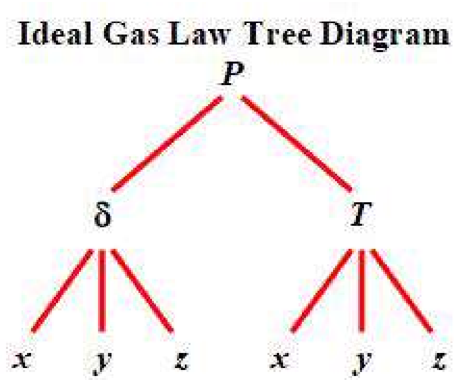
4. The Ideal Gas Law says the Pressure, P, Density, δ, and Temperature, T, are related by $P = k\delta T$.
A particular sample of ideal gas has $k = 2$.
At a certain point the pressure, density and temperature are

$P = 4 \qquad \delta = .01 \qquad T = 200$

The gradients of the density and temperature are

$\vec{\nabla}\delta = \langle .001, .002, .003 \rangle \qquad \vec{\nabla}T = \langle 3, 2, 1 \rangle$

Find the gradient of the pressure.



Hint: Compute each component separately using the chain rule and the tree diagram at the right.

a. $\vec{\nabla}P = \langle .46, .48, 1.22 \rangle$

b. $\vec{\nabla}P = \langle .64, .48, 1.22 \rangle$

c. $\vec{\nabla}P = \langle .46, .84, 1.22 \rangle$

d. $\vec{\nabla}P = \langle .64, .84, 1.22 \rangle$

e. $\vec{\nabla}P = \langle .46, .48, 2.11 \rangle$

f. $\vec{\nabla}P = \langle .64, .48, 2.11 \rangle$

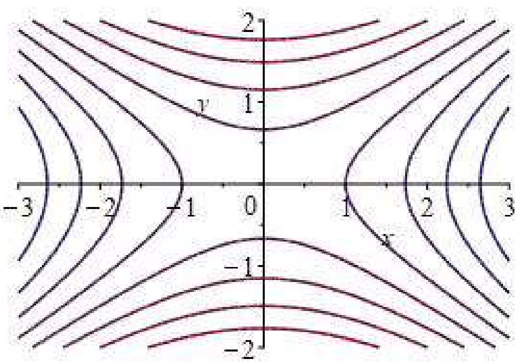
g. $\vec{\nabla}P = \langle .46, .84, 2.11 \rangle$

h. $\vec{\nabla}P = \langle .64, .84, 2.11 \rangle$

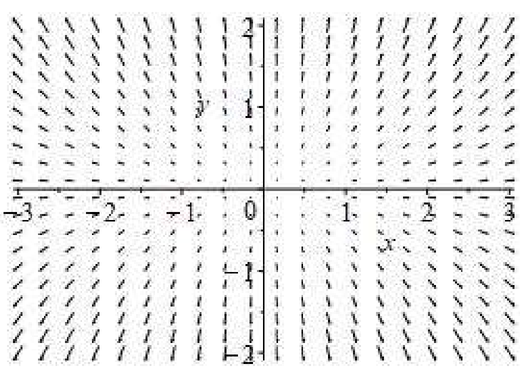
5. The point $(x,y) = (0,2)$ is a critical point of the function $f(x,y) = y^4 - 32y + 8x^2y$.
Use the Second Derivative Test to classify the point or say the test fails.

- **a**. Local Minimum
- **b**. Local Maximum
- **c**. Inflection Point
- **d**. Saddle Point
- **e**. Test FAILS

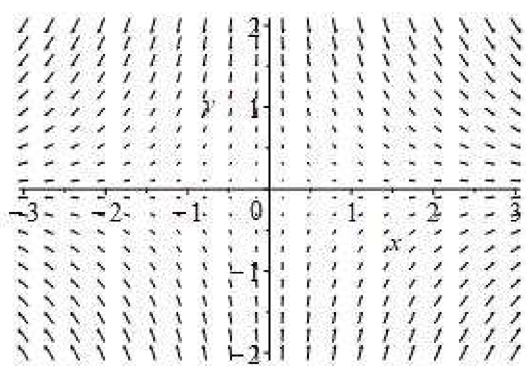
6. The contour plot of a function f is shown. Which of the following is the plot of its gradient, $\vec{\nabla} f$?



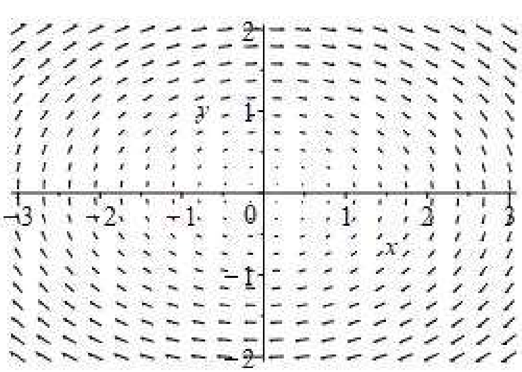
a.



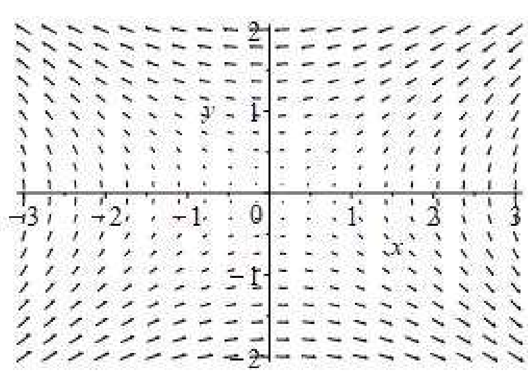
c.



b.



d.



7. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is $\delta = yz + xz + xy$. If Ham's current position is $P = (1,1,3)$, find the rate of change of the density in the direction toward the point $Q = (2,3,1)$.

$$\nabla_{\hat{w}}\delta = \underline{\hspace{3cm}}$$

8. Ham Duet is flying the Centurion Eagle through a nebula where the density of cloaking sparkles is $\delta = yz + xz + xy$. If Ham's current position is $P = (1,1,3)$, in what unit vector direction should he travel to increase the cloaking sparkles as fast as possible?

$$\hat{u} = \langle \underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \underline{\hspace{2cm}} \rangle$$

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (15 points) Consider the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}$.

Determine the value of the limit or show the limit does not exist.

10. (20 points + 5 pts extra credit) Find the largest value of $f = xyz$ on the ellipsoid $\frac{x^2}{16} + \frac{y^2}{4} + z^2 = 3$.

NOTE: Solve by either Eliminating a Variable or by Lagrange Multipliers.
Extra Credit for solving by both methods.
Draw a line across your paper to clearly separate the two solutions.
HINT: When Eliminating a Variable, maximize $F = f^2 = x^2y^2z^2$.

11. (15 points) For each vector field, calculate its divergence and curl. Say if it has a scalar potential or a vector potential. Find the scalar potential if it exists. Do NOT find the vector potential.

a. $\vec{F} = \langle -xy^2, x^2y, y^2z - x^2z \rangle$

$\vec{\nabla} \cdot \vec{F} =$

$\vec{\nabla} \times \vec{F} =$

Has a scalar potential? Yes No Has a vector potential? Yes No

Find a scalar potential:

$f =$ ____________________

b. $\vec{G} = \langle 2xz, 2yz, x^2 + y^2 + 2z \rangle$

$\vec{\nabla} \cdot \vec{G} =$

$\vec{\nabla} \times \vec{G} =$

Has a scalar potential: Yes No Has a vector potential: Yes No

Find a scalar potential:

$g =$ ____________________