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MATH 221 Exam 3 Fall 2021

Sections 504/505 (circle one) P. Yasskin

1-9	/45	11	/20
10	/20	12	/20
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Compute $I = \int_0^4 \int_0^3 \int_0^2 x^3 y^2 z \, dz \, dy \, dx$. Simplify to an integer.

$I =$ ______________

2. Find the mass of the triangle with vertices $(0,-3)$, $(0,3)$ and $(3,0)$ if the density is $\delta = x$. Simplify to an integer.

$M =$ ______________

3. Find the x-component of the center of mass of the triangle with vertices $(0,-3)$, $(0,3)$ and $(3,0)$, if the density is $\delta = x$. Simplify to a rational number. Enter $\frac{7}{5}$ as 7/5.

$\bar{x} =$ ______________

4. Estimate the double integral $I = \iint_R x^2 y \, dA$ over the rectangle $[0,4] \times [0,8]$ using a Riemann sum with 4 small rectangles which are 2 wide and 4 high with evaluation points at the center of each small rectangle.

$I \approx$ ____________

5. Compute the integral $\int_0^1 \int_{\sqrt{y}}^1 x(x^4+1)^{24} \, dx \, dy$.

HINT: Reverse the order of integration.

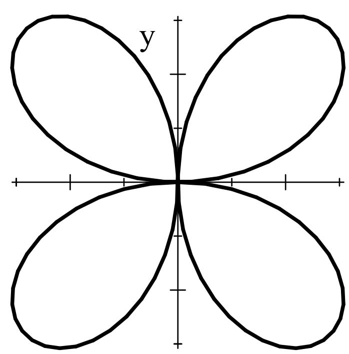
a. $\frac{1}{24}(2^{23}-1)$ **e**. $\frac{1}{6}(2^{23}-1)$ **i**. $\frac{1}{96}(2^{23}-1)$

b. $\frac{1}{25}(2^{25}-1)$ **f**. $\frac{4}{25}(2^{25}-1)$ **j**. $\frac{1}{100}(2^{25}-1)$

c. $\frac{1}{96}(2^{95}-1)$ **g**. $\frac{1}{24}(2^{95}-1)$ **k**. $\frac{1}{384}(2^{95}-1)$

d. $\frac{1}{97}(2^{97}-1)$ **h**. $\frac{4}{97}(2^{97}-1)$ **l**. $\frac{1}{388}(2^{97}-1)$

6. The graph of $r = \sin(2\theta)$ is the 4-leaf clover. Find the area of the leaf in the first quadrant. Enter $\frac{5\pi}{6}$ as 5pi/6.



$A =$ ____________

7. Find the average value of the function $f(x,y,z) = z$ over the solid P below the paraboloid $z = 9 - x^2 - y^2$ and above the xy-plane. Simplify completely.
HINT: Don't use rectangular coordinates.

$f_{\text{ave}} =$ ______________

8. Find the z-component of the centriod of the $\frac{1}{8}$ of a sphere of radius 4 centered at the origin in the first octant (i.e. $x \geq 0$, $y \geq 0$ and $z \geq 0$).

$\bar{z} =$ ______________

9. Compute $\iint_C \vec{\nabla} \cdot \vec{F}\, dS$ over the cylindrical surface $x^2 + y^2 = 4$ for $0 \leq z \leq 3$ if $\vec{F} = \langle xz, yz, z^2 \rangle$.
You can parametrize the surface as $\vec{R}(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z \rangle$.

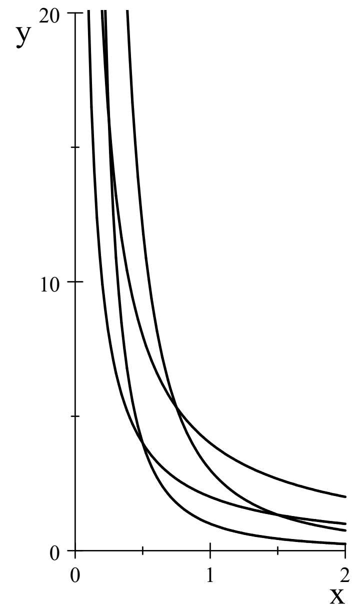
$\iint_C \vec{\nabla} \cdot \vec{F}\, dS =$ ______________

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 points) Compute $\iint_D x^3 y^2 \, dA$ over the diamond shaped region in the first quadrant bounded by the curves

$$y = \frac{2}{x} \qquad y = \frac{4}{x} \qquad y = \frac{1}{x^2} \qquad y = \frac{3}{x^2}$$

HINT: Let $u = xy$ and $v = x^2 y$. What are $\frac{u}{v}$ and $\frac{u^2}{v}$?



11. (20 points) Compute the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = \langle xz, yz, z^2 \rangle$ over the hemisphere $x^2 + y^2 + z^2 = 9$ for $z \geq 0$ with the outward orientation.
The hemisphere may be parametrized as $\vec{R} = \langle 3\sin\phi\cos\theta, 3\sin\phi\sin\theta, 3\cos\phi \rangle$.
HINT: Successively find $\vec{e}_\phi$, $\vec{e}_\theta$, $\vec{N}$, $\vec{F}\big|_{\vec{R}}$ and $\vec{F} \cdot \vec{N}$.

12. (20 points) A spiral ramp may be parametrized by

$$\vec{R}(r,\theta) = \langle r\cos\theta, r\sin\theta, \theta \rangle$$

Find the mass of the spiral ramp for $1 \le r \le 2$ and two turns, i.e. $0 \le \theta \le 4\pi$, if the surface density is given by $\delta = \sqrt{x^2 + y^2}$.