Name________________________

MATH 221 Exam 1 Spring 2023

Section 501 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1-9	/54	11	/25
10	/15	12	/10
		Total	/104

1. Consider the sphere which has a diameter with endpoints $P = (6.-3,-5)$ and $Q = (-2,5,-1)$. Find the center.

a. $(-4,4,2)$

b. $(4,-4,-2)$

c. $(2,1,-3)$

d. $(-2,-1,3)$

e. None of these

2. Consider the sphere which has a diameter with endpoints $P = (6.-3,-5)$ and $Q = (-2,5,-1)$. Find the radius.

a. 6

b. 12

c. 36

d. 144

e. None of these

3. If $\vec{u}$ points Up and $\vec{v}$ points NorthWest, where does $\vec{u}\times\vec{v}$ point?

a. Down

b. SouthWest

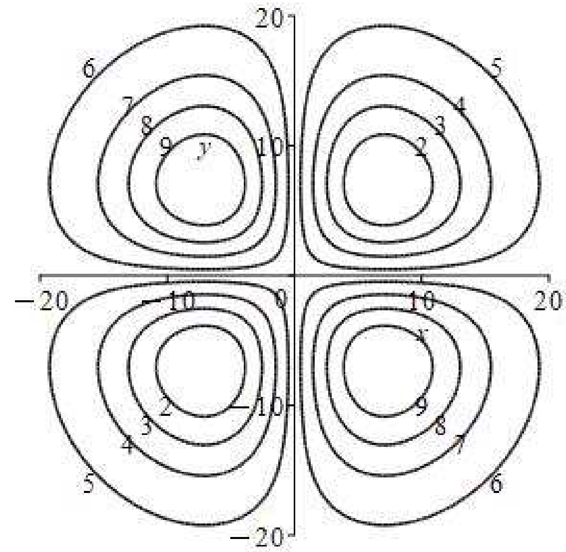
c. SouthEast

d. NorthEast

e. South

4. The figure shows the contour plot of a function $f(x,y)$.
The level sets are labeled by the values of f.
Which point is a local maximum?

a. $(7,0)$
b. $(15,-15)$
c. $(-7,-7)$
d. $(-7,7)$
e. $(0,0)$



5. Consider the contour plot in the previous problem. Which point is a saddle point?

a. $(7,0)$
b. $(15,-15)$
c. $(-7,-7)$
d. $(-7,7)$
e. $(0,0)$

6. Write the vector $\vec{u} = \langle 7,2,3 \rangle$ as the sum of a vector $\vec{p}$ which is parallel to $\vec{v} = \langle 3,2,1 \rangle$ and a vector $\vec{q}$ which is perpenducular to $\vec{v}$. Then $\vec{q} =$

a. $\langle 1,-2,1 \rangle$
b. $\langle -2,2,2 \rangle$
c. $\langle 2,-3,0 \rangle$
d. $\langle -2,4,-2 \rangle$
e. $\langle 1,-1,-1 \rangle$

7. Find a vector perpendicular to the plane containing the points

$$A = (2,0,4), \quad B = (2,1,3) \quad \text{and} \quad C = (3,1,4).$$

a. $\langle 1,1,-1\rangle$

b. $\langle -1,-1,-1\rangle$

c. $\langle 1,1,1\rangle$

d. $\langle -1,1,-1\rangle$

e. $\langle 1,-1,-1\rangle$

8. Write $(4,1,-4)$ as a linear combination of $(2,3,3)$ and $(1,-1,-2)$ or determine that it is impossible.

In other words, find a and b so that

$$(4,1,-4) = a(2,3,3) + b(1,-1,-2)$$

Then $a + 2b =$

a. 3

b. 5

c. 8

d. 12

e. Impossible

9. Classify the surface: $2x^2 - 8x - y^2 + 6y + z^2 = 1$.

a. Hyperbolic Paraboloid

b. Hyperbolic Cylinder

c. Hyperboloid of 1 sheet

d. Hyperboloid of 2 sheets

e. Cone

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (15 points) Find the point where the line $(x,y,z) = (2-t, 1+2t, 3-t)$ intersects the plane $3x + 2y - 3z = 11$.

$(x,y,z) =$ ______________________

11. (25 points) For the curve $\vec{r}(t) = \langle e^{2t}, 2e^t, t \rangle$ compute each of the following:

a. (5 pts) The velocity $\vec{v}$

$\vec{v} =$ ______________________

b. (5 pts) The speed $\frac{ds}{dt}$ (Simplify!)

$\frac{ds}{dt} =$ ______________________

c. (5 pts) The tangential acceleration a_T

$a_T =$ ______________________

d. (5 pts) The length of this curve between $(1,2,0)$ and $(e^2, 2e, 1)$.

$L =$ ______________________

e. (5 pts) The unit binormal vector $\hat{B}$

$\hat{B} =$ ____________________________________

12. (10 points) The volume of a pyramid is

$$V = \frac{1}{3}BH$$

where B is the area of the base and H is the height. Derive a formula for the volume of the triangular pyramid with edge vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Your formula should involve the dot, cross and/or triple product of $\vec{u}$, $\vec{v}$ and $\vec{w}$. Your derivation should explain all steps like the book does for the area of a triangle or the volume of a parallelepiped. Use sentences.