Name____________________________ Section:__________

MATH 221 Exam 1, Version A Fall 2023

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Multiple Choice: (6 points each. No part credit.)

1-9	/54	12	/10
10	/20	13	/10
11	/10	Total	/104

1. The circle $(x-2)^2 + (y-5)^2 = 9$ is tangent to which line?
HINT: Draw a picture.

a. $x = 1$

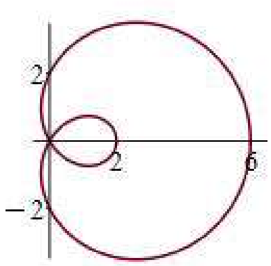
b. $x = 3$

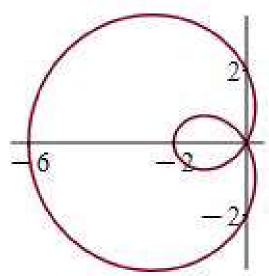
c. $x = 5$

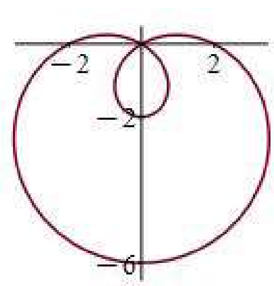
d. $y = 1$

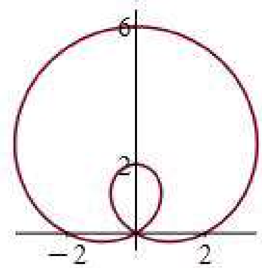
e. $y = 4$

2. Which of the following is the plot of the polar curve $r = 2 - 4\cos\theta$?

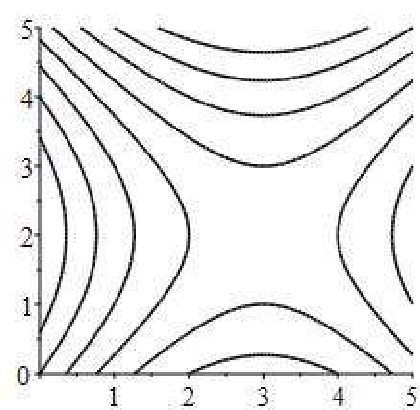
a. 

b. 

c. 

d. 

3. The plot at the right is the contour plot of which function?



a. $z = (x-2)^2 + (y-3)^2$

b. $z = (x-2)^2 - (y-3)^2$

c. $z = (x-3)^2 + (y-2)^2$

d. $z = (x-3)^2 - (y-2)^2$

4. The force $\vec{F} = \langle 5,2 \rangle$ pushes a mass from $P = (5,4)$ to $Q = (12,1)$.
Find the angle between the force and the displacement.

- **a**. $30°$
- **b**. $45°$
- **c**. $60°$
- **d**. $120°$
- **e**. $135°$

5. Find the area of the triangle with vertices $A = (1,2,3)$, $B = (4,6,4)$ and $C = (4,6,6)$.

- **a**. 1
- **b**. 4
- **c**. 5
- **d**. 8
- **e**. 10

6. Find an equation of the line through the point $P = (2,3,4)$
which is perpendicular to the plane $4x + 3y + 2z = 15$.
Then find the point where the line passes through the xy-plane.

- **a**. $(x,y,z) = (10,9,0)$
- **b**. $(x,y,z) = (-10,-9,0)$
- **c**. $(x,y,z) = (-6,-3,0)$
- **d**. $(x,y,z) = (6,3,0)$
- **e**. $(x,y,z) = (6,6,0)$

7. Classify the quadratic surface: $-2x^2 + 4x + 3y^2 + 6y - z + 3 = 0$

a. elliptic paraboloid opening up in the z-direction
b. elliptic paraboloid opening down in the z-direction
c. hyperbolic paraboloid opening up in the x-direction and down in the y-direction
d. hyperbolic paraboloid opening up in the y-direction and down in the x-direction
e. hyperbolic cylinder

8. If an airplane is flying from West to East directly above the equator, where does $\vec{B}$ point? Why?

a. North
b. South
c. West
d. Up
e. Down

9. Find the circulation in a bowl of water, counterclockwise around the circle $x^2 + y^2 = 9$, with $z = 2$, if its fluid velocity field is $\vec{V} = \langle 2x - y, x + 2y, -z \rangle$.

a. 3π
b. 6π
c. 12π
d. 15π
e. 18π

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 pts) Consider the twisted cubic $\vec{r} = (6t, 3t^2, t^3)$. Compute each of the following.
Note: $4 + 4t^2 + t^4 = (2 + t^2)^2$

a. (6 pts) Arc length between $(0,0,0)$ and $(6,3,1)$.

b. (6 pts) Curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$.
HINT: Factor out an 18^2.

c. (4 pts) Tangential acceleration, a_T.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

d. (4 pts) Normal acceleration, a_N.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

11. (10 pts) Find the y-component of the center of mass of the semicircle $y = \sqrt{9 - x^2}$ if its linear density is $\delta(x,y) = y$.
HINT: The semicircle may be parametrized by $\vec{r}(t) = (3\cos t, 3\sin t)$ for $0 \le t \le \pi$.

12. (10 pts) Write the vector, $\langle 1,1,4\rangle$, as a linear composition of $\langle 2,1,3\rangle$ and $\langle 3,1,2\rangle$, i.e. find a and b so that:

$$\langle 1,1,4\rangle = a\langle 2,1,3\rangle + b\langle 3,1,2\rangle$$

or show it cannot be done.

13. (10 pts) Consider the line and the plane:

$$L: \quad \frac{x-2}{2} = \frac{y-3}{2} = \frac{z-1}{2}$$

$$P: \quad 4x - 3y + z = 4$$

Determine if they are parallel or intersecting. If they intersect, find the point of intersection.
You MUST show why they are or are not parallel.