Name______________________________ Section:__________

MATH 221 Exam 1, Version C Fall 2023

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1-9	/54	12	/10
10	/20	13	/10
11	/10	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. The circle $(x-2)^2+(y-5)^2=9$ is tangent to which line?
HINT: Draw a picture.

a. $x=1$

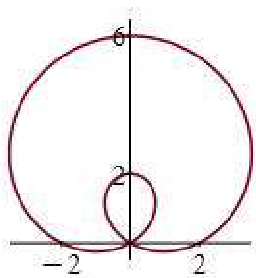
b. $x=3$

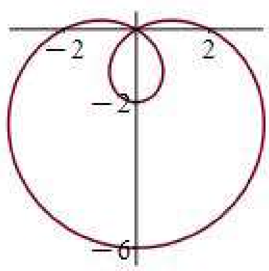
c. $x=5$

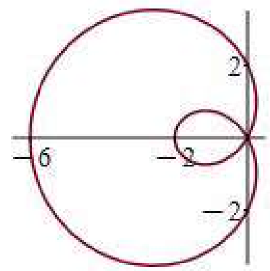
d. $y=1$

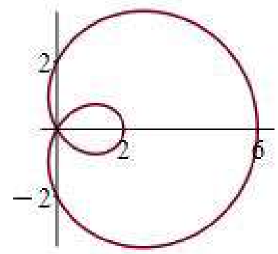
e. $y=4$

2. Which of the following is the plot of the polar curve $r=4\cos\theta-2$?

a. 

b. 

c. 

d. 

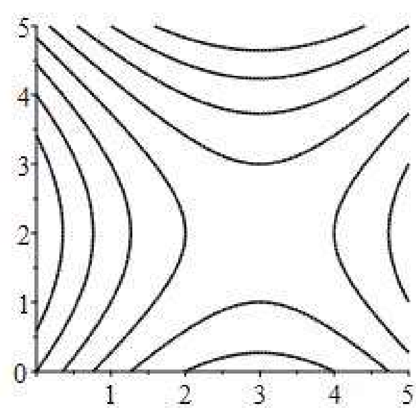
3. The plot at the right is the contour plot of which function?

a. $z=(x-2)^2+(y-3)^2$

b. $z=(x-2)^2-(y-3)^2$

c. $z=(x-3)^2+(y-2)^2$

d. $z=(x-3)^2-(y-2)^2$



4. At a certain point on a certain curve, $\vec{T} = \frac{1}{\sqrt{14}}\langle 3,2,-1\rangle$ and $\vec{B} = \frac{1}{\sqrt{21}}\langle 2,-1,4\rangle$. Find $\vec{N}$.

a. $\vec{N} = \frac{1}{\sqrt{6}}\langle -1,2,-1\rangle$
b. $\vec{N} = \frac{1}{\sqrt{6}}\langle -1,2,1\rangle$
c. $\vec{N} = \frac{1}{\sqrt{6}}\langle 1,2,-1\rangle$
d. $\vec{N} = \frac{1}{\sqrt{6}}\langle 1,-2,-1\rangle$
e. $\vec{N} = \frac{1}{\sqrt{6}}\langle -1,-2,1\rangle$

5. Find the area of the triangle with vertices $A - (1,1,1)$, $B = (5,1,3)$ and $C = (3,2,1)$.

a. 1
b. 3
c. 6
d. 12
e. 18

6. Find an equation of the line through the point $P = (2,3,4)$
which is perpendicular to the plane $4x + 3y + 2z = 15$.
Then find the point where the line passes through the xy-plane.

a. $(x,y,z) = (10,9,0)$
b. $(x,y,z) = (-10,-9,0)$
c. $(x,y,z) = (-6,-3,0)$
d. $(x,y,z) = (6,3,0)$
e. $(x,y,z) = (6,6,0)$

7. Classify the quadratic surface: $2x^2 - 8x + y^2 - 6y - z^2 + 2z = -17$.

a. elliptic paraboloid opening up in the z direction
b. elliptic paraboloid opening down in the z direction
c. hyperboloid of 1-sheet
d. hyperboloid of 2-sheets
e. cone

8. If an airplane is flying from East to West directly above the equator, where does $\vec{B}$ point? Why?

a. North
b. South
c. West
d. Up
e. Down

9. Find the work done by the force $\vec{F} = \langle z^2, yz, xz \rangle$ to move a bead along the twisted cubic $\vec{r}(t) = (t, t^2, t^3)$ from $t = 0$ to $t = 1$.

a. 1
b. $\frac{2}{7}$
c. $\frac{3}{7}$
d. $\frac{6}{7}$
e. $\frac{12}{7}$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 pts) Consider the twisted cubic $\vec{r} = (3t, 3t^2, 2t^3)$. Compute each of the following.
Note: $1 + 4t^2 + 4t^4 = (1 + 2t^2)^2$

a. (6 pts) Arc length between $(0,0,0)$ and $(3,3,2)$.

b. (6 pts) Curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|}$.
HINT: Factor out an 18^2.

c. (4 pts) Tangential acceleration, a_T.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

d. (4 pts) Normal acceleration, a_N.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

11. (10 pts) Find the average value of the function $f(x,y,z) = y^2$
on the helix $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ for $0 \le t \le 2\pi$.

12. (10 pts) Write the vector, $\langle 1,1,3\rangle$, as a linear composition of $\langle 2,1,3\rangle$ and $\langle 3,1,2\rangle$,
i.e. find a and b so that:

$$\langle 1,1,3\rangle = a\langle 2,1,3\rangle + b\langle 3,1,2\rangle$$

or show it cannot be done

13. (10 pts) Consider the 2 lines:

$$L_1(t): \qquad (x,y,z) = (7+2t, 4+t, 3+t)$$

$$L_2(t): \qquad (x,y,z) = (5-2t, 1+t, -2+3t)$$

Determine if they are parallel, intersecting or skew. If they intersect, find the point of intersection. You MUST show why they are or are not parallel or skew.