Name________________________________ Section:__________

MATH 221 Exam 2, Version A Fall 2023

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1-8	/48	10	/20
9	/36	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Consider the function $z = f(x,y) = xy^3$. Its y-trace with $x = 2$ is the intersection of the graph of $z = f(x,y)$ and the plane $x = 2$. Find the slope of this y-trace at $y = 3$.

a. 8

b. 24

c. 27

d. 54

e. 96

2. Consider the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^4 + y^2}$.

Which of the following paths of approach gives a different value of the limit?

a. $y = 2x$ & $x \to 0$

b. $y = x^2 + x^3$ & $x \to 0$

c. $y = x^2 + x$ & $x \to 0$

d. $y = x^2$ & $x \to 0$

e. They are all equal.

Hint: Don’t bother multiplying out any quadratic.

3. Find the plane tangent to the graph of the function $z = x^2 e^{(2y-4)}$ at the point $(3,2)$. Its z-intercept is

a. $c = -45$

b. $c = -33$

c. $c = -9$

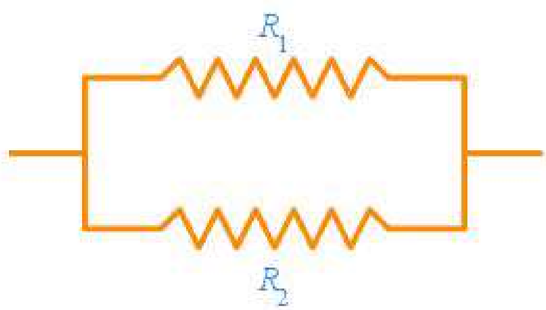
d. $c = 9$

e. $c = 33$

4. If two resistors, with resistances R_1 and R_2 are connected in parallel, then the total resistance R satisfies:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

Currently $R_1 = 200$ ohms and $R_2 = 400$ ohms. If they increase by $\Delta R_1 = 1.8$ ohms and $\Delta R_2 = 0.9$ ohms, use the linear approximation to approximate how much R changes.



a. $\Delta R \approx 0.1$ ohms

b. $\Delta R \approx 0.3$ ohms

c. $\Delta R \approx 0.6$ ohms

d. $\Delta R \approx 0.9$ ohms

e. $\Delta R \approx 1.8$ ohms

5. Let $f(x,y) = x\sin(xy)$. Compute $f_{xy}\left(2, \frac{\pi}{8}\right)$.

a. $2\sqrt{2} + \frac{1}{4}\sqrt{2}\,\pi$

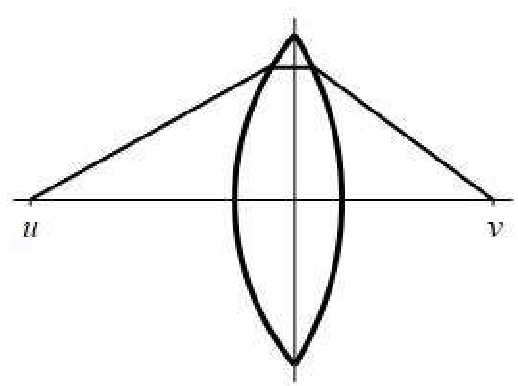
b. $2\sqrt{2} - \frac{1}{4}\sqrt{2}\,\pi$

c. $-2\sqrt{2} + \frac{1}{4}\sqrt{2}\,\pi$

d. $-2\sqrt{2} - \frac{1}{4}\sqrt{2}\,\pi$

6. If the focal length of a lens is f and an object is placed a distance u from the lens, then the image of the object will appear to be a distance v from the lens related by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \text{or} \qquad v = \frac{fu}{u-f}$$



Consider a lens with an adjustable focal length. Currently, $f = 2$ cm, $u = 8$ cm and $v = \frac{8}{3}$ cm.

If f and u are increasing at $\frac{df}{dt} = 0.9\ \frac{\text{cm}}{\text{sec}}$ and $\frac{du}{dt} = 1.8\ \frac{\text{cm}}{\text{sec}}$, at what rate is v changing?

a. $\frac{dv}{dt} = 1\ \frac{\text{cm}}{\text{sec}}$

b. $\frac{dv}{dt} = 1.2\ \frac{\text{cm}}{\text{sec}}$

c. $\frac{dv}{dt} = 1.4\ \frac{\text{cm}}{\text{sec}}$

d. $\frac{dv}{dt} = 1.6\ \frac{\text{cm}}{\text{sec}}$

e. $\frac{dv}{dt} = 1.8\ \frac{\text{cm}}{\text{sec}}$

7. Find the equation of the plane tangent to $x^2z^2 - yz = 3$ at the point $P = (1,2,3)$.
It's z-intercept is:

a. $c = 3$

b. $c = 4$

c. $c = 6$

d. $c = 12$

e. $c = 24$

8. The point $(x,y) = (1,1)$ is a critical point of the function $f(x,y) = 3x^2 - 6xy - 2y^3 + 6y^2$.
Use the Second Derivative Test to classify this critical point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (36 pts) Obi-Two is flying the Centurion Eagle through the Force, Desperation and Luck fields. The Force, F, depends on the Desperation, D, and Luck, L, by the relation: $F = D^3 L$. Currently, the Desperation and Luck and their gradients are:

$$D = 2 \qquad \vec{\nabla} D = \langle 0,1,2 \rangle$$
$$L = 3 \qquad \vec{\nabla} L = \langle 3,1,0 \rangle$$

a. (23 pts) Obi-Two's current velocity is $\vec{v} = \langle 2,1,3 \rangle$. Find the rate that Obi-Two sees the Force changing.

b. (13 pts) In what direction should Obi-Two travel to increase the Force as fast as possible? HINT: Compute each (x, y, z) partial derivative separately.

10. (20 pts) Find the largest value of the function $f = xy^2z^4$ on the ellipsoid:

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{2} = \frac{7}{2}$$