Name________________________________ Section:__________

MATH 221 Exam 2, Version B Fall 2023

502,503 P. Yasskin

1-8	/48	10	/20
9	/36	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Consider the function $z = f(x,y) = xy^3$. Its x-trace with $y = 3$ is the intersection of the graph of $z = f(x,y)$ and the plane $y = 3$. Find the slope of this x-trace at $x = 2$.

a. 8

b. 24

c. 27

d. 54

e. 96

2. Consider the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2}$.

Which of the following paths of approach gives a different value of the limit?

a. $y = 2x$ & $x \to 0$

b. $y = x^2 + x^3$ & $x \to 0$

c. $y = x^2 + x$ & $x \to 0$

d. $y = x^3$ & $x \to 0$

e. They are all equal.

Hint: Don't bother multiplying out any quadratic.

3. Find the plane tangent to the graph of the function $z = x\cos y$ at the point $\left(\sqrt{2}, \frac{\pi}{4}\right)$.
Its z-intercept is

a. $c = \frac{\pi}{4}$

b. $c = -\frac{\pi}{4}$

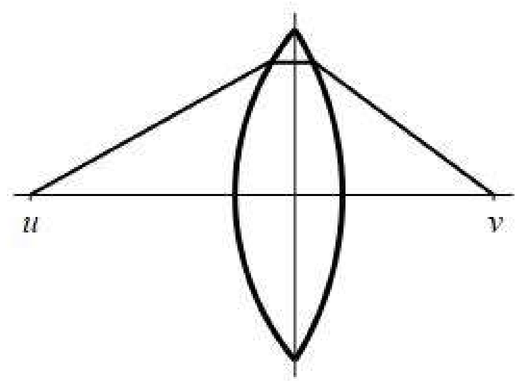
c. $c = \frac{\pi}{4}\sqrt{2}$

d. $c = -\frac{\pi}{2}\sqrt{2}$

e. $c = 1 + \sqrt{2} - \frac{\pi}{4}$

4. If the focal length of a lens is f and an object is placed a distance u from the lens, then the image of the object will appear to be a distance v from the lens related by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \text{or} \qquad v = \frac{fu}{u-f}$$



Consider a lens with an adjustable focal length.
Currently, $f = 2$ cm, $u = 8$ cm and $v = \frac{8}{3}$ cm.
If f and u increase by $\Delta f = 0.9$ cm and $\Delta u = 1.8$ cm, use the linear approximation to approximate how much v changes.

a. $\Delta v \approx 1$ cm

b. $\Delta v \approx 1.2$ cm

c. $\Delta v \approx 1.4$ cm

d. $\Delta v \approx 1.6$ cm

e. $\Delta v \approx 1.8$ cm

5. Let $f(x,y) = x\cos(xy)$. Compute $f_{xy}\left(2, \frac{\pi}{8}\right)$.

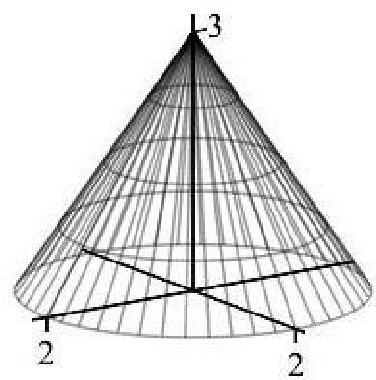
a. $2\sqrt{2} + \frac{1}{4}\sqrt{2}\,\pi$

b. $2\sqrt{2} - \frac{1}{4}\sqrt{2}\,\pi$

c. $-2\sqrt{2} + \frac{1}{4}\sqrt{2}\,\pi$

d. $-2\sqrt{2} - \frac{1}{4}\sqrt{2}\,\pi$

6. The volume of a cone is $V = \frac{1}{12}\pi D^2 H$ where D is the diameter and H is the height.
Currently, $D = 4$ cm, $H = 3$ cm and $V = 4\pi$ cm.
If V and H are increasing at $\frac{dV}{dt} = 0.6\pi\ \frac{\text{cm}}{\text{sec}}$ and $\frac{dH}{dt} = 0.3\ \frac{\text{cm}}{\text{sec}}$, at what rate is D changing?



a. $\frac{dD}{dt} = 0.1\ \frac{\text{cm}}{\text{sec}}$

b. $\frac{dD}{dt} = 0.25\ \frac{\text{cm}}{\text{sec}}$

c. $\frac{dD}{dt} = 0.5\ \frac{\text{cm}}{\text{sec}}$

d. $\frac{dD}{dt} = 0.6\ \frac{\text{cm}}{\text{sec}}$

e. $\frac{dD}{dt} = 0.75\ \frac{\text{cm}}{\text{sec}}$

7. Find the equation of the plane tangent to $x^2y^2e^{2z-4} = 1$ at the point $P = (1,1,2)$.
It's z-intercept is:

a. $c = 0$

b. $c = 1$

c. $c = 2$

d. $c = 3$

e. $c = 4$

8. The point $(x,y) = (1,2)$ is a critical point of the function $f(x,y) = 2x^3y^2 + x^2y^3 - 5x^2y^2$.
Use the Second Derivative Test to classify this critical point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (36 pts) Obi-Two is flying the Centurion Eagle through the Force, Desperation and Luck fields. The Force, F, depends on the Desperation, D, and Luck, L, by the relation: $F = 2D^2L$. Currently, the Desperation and Luck and their gradients are:

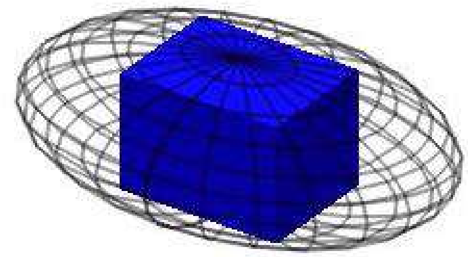
$$D = 2 \qquad \vec{\nabla}D = \langle 1, 0, 3 \rangle$$

$$L = 3 \qquad \vec{\nabla}L = \langle 2, 1, 0 \rangle$$

a. (23 pts) Obi-Two's current velocity is $\vec{v} = \langle 3, 2, 1 \rangle$. Find the rate that Obi-Two sees the Force changing.

b. (13 pts) In what direction should Obi-Two travel to increase the Force as fast as possible? HINT: Compute each (x, y, z) partial derivative separately.

10. (20 pts) Find the volume of the largest rectangular solid with its 8 vertices on the ellipsoid:



$$\frac{x^2}{8} + \frac{y^2}{18} + \frac{z^2}{2} = 6$$