Name________________________________ Section:__________

MATH 221 Exam 3, Version A Fall 2023

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1-8	/48	10	/16
9	/16	11	/24
		Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Find the divergence of the vector field $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$ and evaluate the divergence at $P = (1,2,3)$.

a. -2

b. 20

c. 22

d. 23

e. 24

2. Find the curl of the vector field $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$ and evaluate the curl at $P = (1,2,3)$.

a. $\langle -4, -9, -1 \rangle$

b. $\langle -4, 9, -1 \rangle$

c. $\langle 2, -12, 9 \rangle$

d. $\langle 2, 12, 9 \rangle$

e. $\langle 1, -4, 9 \rangle$

3. Let f be a scalar potential for $\vec{F} = \langle yz + 2x, xz + 2y, xy + 2z \rangle$. Compute $f(1,2,3) - f(0,0,0)$.
(Note: The subtraction cancels off the arbitrary constant.)

a. -2

b. 20

c. 22

d. 23

e. 24

4. Use a Riemann sum with 6 squares evaluated at the center of each square to estimate the volume of the solid over the rectangle $[1,7]\times[2,6]$ below the surface $f = x^2 + y^2$.

a. 130

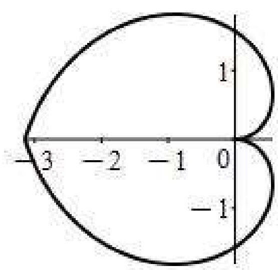
b. 214

c. 520

d. 856

e. 872

5. Find the area inside the heart which in polar coordinates is the spiral $r = |\theta|$ for $-\pi \le \theta \le \pi$.
HINT Double the area inside half the spiral.



a. $\dfrac{\pi^3}{6}$

b. $\dfrac{\pi^3}{3}$

c. $\dfrac{\pi^3}{2}$

d. $\dfrac{\pi^4}{4}$

e. $\dfrac{\pi^4}{2}$

6. Find mass of the solid below $z = 25 - x^2 - y^2$ above the xy-plane inside the cylinder $x^2 + y^2 = 9$ if the volume density is $\delta = z$.

a. $\frac{\pi}{6}(25^3 - 16^3)$

b. $\frac{\pi}{6}(25^3 + 16^3)$

c. $\pi\left(25^2 \cdot 3 - 50 \cdot 3^2 + \frac{3^5}{5}\right)$

d. $\pi\left(25^2 \cdot 3 + 50 \cdot 3^2 + \frac{3^5}{5}\right)$

e. $\pi\left(25\frac{3^2}{2} - \frac{3^4}{4}\right)$

7. Find the Jacobian factor for the 3D coordinate system:

$$(x,y,z) = \vec{R}(u,v,w) = (vw, uw, uv) \qquad \text{with} \quad u > 0, \quad v > 0, \quad w > 0$$

a. uvw

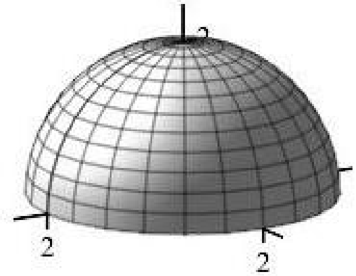
b. $2uvw$

c. $u + v + w$

d. $2u + 2v + 2w$

e. $vw + uw + uv$

8. Find the average value of the function $f = x^2 + y^2 + z^2$ on the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.



a. $\frac{5}{5}$

b. $\frac{4}{5}\pi$

c. $\frac{12}{5}$

d. $\frac{16}{3}\pi$

e. $\frac{64}{5}\pi$

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (16 points) Consider a plate bounded by $y = 4 - x^2$ and the x-axis with surface density $\delta = x^2$.

a. (8 pts) Find the mass of a plate.

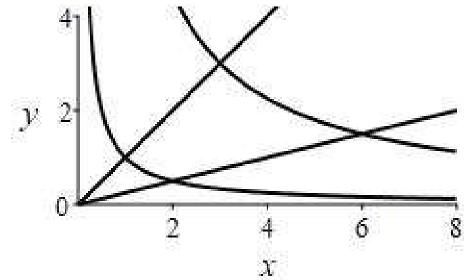
b. (8 pts) Find the center of mass of a plate.

10. (16 points) Compute $\iint_D x^2\, dA$ over the diamond shaped region in the 1^{st} quadrant bounded by

$y = \frac{1}{x}$ $\quad y = \frac{9}{x}$ $\quad y = x$ $\quad y = \frac{1}{4}x$

HINT: Use the curvilinear coordinates

$x = uv$ $\qquad y = \frac{v}{u}$.



a. (5 pts) Find the Jacobian factor.

b. (1 pts) Express the integrand in terms of the coordinates.

c. (4 pts) Substitute $x = uv$ and $y = \frac{v}{u}$ into the boundaries to express them in terms of u and v.

d. (6 pts) Compute the integral.

11. (24 points) Consider the cone surface $z = 2\sqrt{x^2 + y^2}$ for $z \le 6$
which may be parametrized by

$$\vec{R}(r,\theta) = \langle r\cos\theta, r\sin\theta, 2r \rangle$$

a. (6 pts) Find the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

b. (3 pts) Find the normal vector oriented down and out:

$\vec{N} =$

c. (2 pts) Find the length of the normal vector:

$\left|\vec{N}\right| =$

d. (4 pts) Find the mass of the cone if the mass density is $\delta = \sqrt{x^2 + y^2}$.

$M =$

e. (3 pts) Find the **curl** of the vector field $\vec{F} = \langle yz, -xz, z^2 \rangle$ in rectangular coordinates:

$\vec{\nabla} \times \vec{F} =$

f. (2 pts) Evaluate the **curl** of $\vec{F}$ on the cone:

$\vec{\nabla} \times \vec{F}\Big|_{\vec{R}} =$

g. (4 pts) Find the flux of the **curl** of $\vec{F}$ down and out of the cone:

$\iint_C \left(\vec{\nabla} \times \vec{F}\right) \cdot d\vec{S} =$