Name________________________

MATH 221 Exam 2 Version H Fall 2019

Section 204 P. Yasskin

1-8	/48	11	/16
9	/10	12	/25
10	/ 5	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Find the equation of the plane tangent to $z = x^3y + xy^2$ at the point $(x,y) = (1,2)$.
Its z-intercept is:

a. $c = -14$

b. $c = -12$

c. $c = -6$

d. $c = 6$

e. $c = 14$

2. The volume of a frustum of a cone is $V = \frac{\pi}{3}(R^2 + Rr + r^2)h$ where R is the bottom radius, r is the top radius and h is the height. Currently, $R = 2$ cm, $r = 1$ cm and $h = 3$ cm. Use differentials to estimate the change in volume if R and r increase by 0.1 cm while h decreases by 0.3.

a. $\Delta V \approx 3.2\pi$

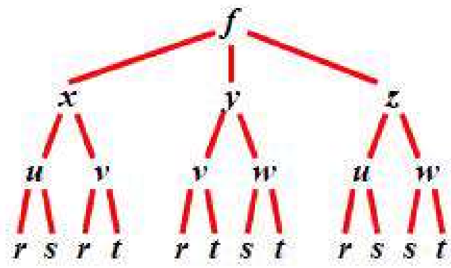
b. $\Delta V \approx 1.6\pi$

c. $\Delta V \approx 0.8\pi$

d. $\Delta V \approx 0.6\pi$

e. $\Delta V \approx 0.2\pi$

3. At the right is a tree diagram showing f as a function of x, y and z which are functions of u, v and w which are functions of r, s and t as indicated.
Below are values of a bunch partial derivatives.
Use this information to compute $\dfrac{\partial f}{\partial r}$.



$$\frac{\partial f}{\partial x} = 2 \qquad \frac{\partial f}{\partial y} = 3 \qquad \frac{\partial f}{\partial z} = 4$$

$$\frac{\partial x}{\partial u} = 5 \qquad \frac{\partial x}{\partial v} = 6 \qquad \frac{\partial y}{\partial v} = 7 \qquad \frac{\partial y}{\partial w} = 8 \qquad \frac{\partial z}{\partial u} = 9 \qquad \frac{\partial z}{\partial w} = 10$$

$$\frac{\partial u}{\partial r} = 6 \qquad \frac{\partial u}{\partial s} = 5 \qquad \frac{\partial v}{\partial r} = 4 \qquad \frac{\partial v}{\partial t} = 3 \qquad \frac{\partial w}{\partial s} = 2 \qquad \frac{\partial w}{\partial t} = 1$$

a. 163

b. 212

c. 358

d. 396

e. 408

4. The point $(x,y) = (-1,2)$ is a critical point of the function $f = 8x^3 - y^3 - 12xy$.
Use the 2^{nd} Derivative Test to classify it as:

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. The 2^{nd} Derivative Test FAILS.

5. If x, y and z are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial x}$ at the point $(x,y,z) = \left(\sqrt{3}, \frac{\pi}{6}, 3\right)$.

a. $\frac{1}{\sqrt{3}}$

b. $\frac{-1}{\sqrt{3}}$

c. $\sqrt{3}$

d. $-\sqrt{3}$

e. $\frac{1}{3}$

6. If x, y and z are related by $x\cos y + z\sin y = 3$. Find $\frac{\partial z}{\partial t}$ at the instant when:

$$(x,y,z) = \left(\sqrt{3}, \frac{\pi}{6}, 3\right) \qquad \frac{dx}{dt} = \frac{1}{\sqrt{3}} \qquad \frac{dy}{dt} = \frac{1}{\sqrt{3}}$$

a. -1

b. -2

c. -3

d. $-\sqrt{3}$

e. $\frac{-1}{\sqrt{3}}$

7. Find the tangent plane to the graph of the equation $xy - zy = -4$ at the point $(x,y,z) = (1,2,3)$.
Its z-intercept is:

a. $c = -8$

b. $c = -4$

c. $c = 0$

d. $c = 4$

e. $c = 8$

8. Queen Lena is flying the Centurion Eagle through a deadly Sythion field whose density is $S = xyz \dfrac{\text{Sythions}}{\text{microlightyear}^3}$. The top speed of the Centurion Eagle is $14 \dfrac{\text{microlightyears}}{\text{lightyear}}$. If Lena is located at the point $(x, y, z) = (3, 2, 1)$, what should her velocity be to **decrease** the Sythion density as fast as possible?

a. $\langle -4, -6, -12 \rangle$

b. $\langle -2, -3, -6 \rangle$

c. $\langle -28, 42, -84 \rangle$

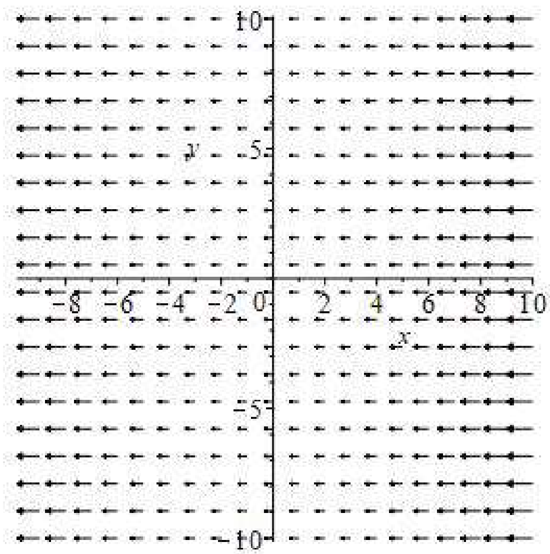
d. $\langle 4, 6, 12 \rangle$

e. $\langle 2, 3, 6 \rangle$

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (10 points) Prove whether the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^6 + y^6}{x^4 + 2x^2y^2 + y^4}$ converges or diverges. If it converges, find the limit.

10. (5 points) Here is the plot of a vector field $\vec{F}$ in $\mathbb{R}^2$.
Shade in the region where $\vec{\nabla} \cdot \vec{F} > 0$. Explain why.



11. (16 points) Let $\vec{F} = \langle x, 2y, -3z \rangle$.

a. Find a scalar potential, f, for $\vec{F}$ or show one does not exist.

b. Find a vector potential, $\vec{A}$, for $\vec{F}$ or show one does not exist.
Explain all steps neatly and clearly.

12. (25 points) Find the largest and smallest values of the function $f(x,y,z) = xyz$ on the ellipsoid $x^2 + 4y^2 + 9z^2 = 108.$