Name________________________ Sec________

1-13	/65
14	/25
15	/15
Total	/105

MATH 251 Final Exam Spring 2008

Sections 508 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Which of the following lines lies in the plane: $2x - y - z = 0$?

a. $(x,y,z) = (1,2,3) + t(1,1,1)$

b. $(x,y,z) = (3,2,1) + t(1,1,1)$

c. $(x,y,z) = (2,1,3) + t(1,1,1)$

d. $(x,y,z) = (3,1,2) + t(1,1,1)$

e. $(x,y,z) = (1,3,2) + t(1,1,1)$

2. Find the equation of the plane tangent to the graph of the function $f(x,y) = x^2y + xy^2$ at the point $(2,1)$. Then the z-intercept is

a. -12

b. -6

c. 0

d. 6

e. 12

3. Find the arc length of the curve $\vec{r}(t) = (e^t, 2t, 2e^{-t})$ between $(1,0,2)$ and $(e,2,2e^{-1})$.

Hint: Look for a perfect square.

a. $e - 2e^{-1}$

b. $1 + e - 2e^{-1}$

c. $e - 2e^{-1} - 1$

d. $e + 2e^{-1}$

e. $e + 2e^{-1} - 3$

4. Find the tangential acceleration a_T of the curve $\vec{r}(t) = (e^t, 2t, 2e^{-t})$.

Hint: Which formula is easier?

a. $e^t + 2e^{-t}$

b. $e^t + 4e^{-t}$

c. $e^t - 2e^{-t}$

d. $e^t - 4e^{-t}$

e. $e^{2t} - 4e^{-2t}$

5. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.
If the radius r is currently 3 cm and increasing at 2 cm/sec
while the height h is currently 4 cm and decreasing at 1 cm/sec,
is the volume increasing or decreasing and at what rate?

a. decreasing at 19π cm$^3/\text{sec}$

b. decreasing at 13π cm$^3/\text{sec}$

c. neither increasing nor decreasing

d. increasing at 13π cm$^3/\text{sec}$

e. increasing at 19π cm$^3/\text{sec}$

6. Which of the following is a local maximum of $f(x,y) = \sin(x)\sin(y)$?

a. $(0,0)$

b. $\left(\frac{\pi}{2},0\right)$

c. (π,π)

d. $\left(0,\frac{\pi}{2}\right)$

e. None of the above

7. Find the equation of the plane tangent to the surface $\frac{x^2}{z^2} + \frac{y^3}{z^3} = 17$ at the point $(3,2,1)$.

Then the z-intercept is

a. 0

b. -42

c. 42

d. -18

e. 18

8. Compute $\int \vec{F} \cdot d\vec{s}$ counterclockwise around the circle $x^2 + y^2 = 4$

for the vector field $\vec{F} = \left(-x^4 y + \frac{1}{3}x^2 y^3, xy^4 + x^3 y^2\right)$.

Hint: Use Green's Theorem and factor the integrand.

a. $\frac{4\pi}{3}$

b. $\frac{8\pi}{3}$

c. $\frac{16\pi}{3}$

d. $\frac{32\pi}{3}$

e. $\frac{64\pi}{3}$

9. Find the mass of the half cylinder $x^2 + y^2 \leq 4$ for $0 \leq z \leq 10$ and $y \geq 0$ if the density is $\rho = x^2 + y^2$.

a. 10π

b. 20π

c. 40π

d. 80π

e. 160π

10. Find the center of mass of the half cylinder $x^2 + y^2 \leq 4$ for $0 \leq z \leq 10$ and $y \geq 0$ if the density is $\rho = x^2 + y^2$.

a. $(0, 1, 5)$

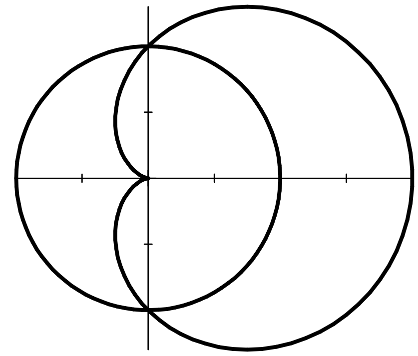
b. $\left(0, \frac{8}{5\pi}, 5\right)$

c. $\left(0, \frac{5\pi}{8}, 5\right)$

d. $\left(0, \frac{16}{5\pi}, 5\right)$

e. $\left(0, \frac{5\pi}{16}, 5\right)$

11. Find the area inside the cardioid $r = 1 + \cos\theta$ but outside the circle $r = 1$.



a. $\frac{\pi}{4}$

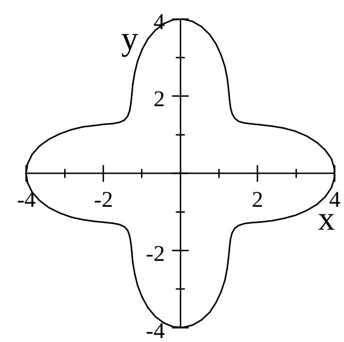
b. $\frac{\pi}{2}$

c. $2 - \frac{\pi}{4}$

d. $2 + \frac{\pi}{4}$

e. $2 - \frac{\pi}{2}$

12. Compute $\oint \vec{\nabla} f \cdot d\vec{s}$ counterclockwise once around the polar curve $r = 3 + \cos(4\theta)$ for the function $f(x,y) = x^2 y$.



a. 2π

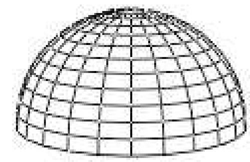
b. 4π

c. 6π

d. 8π

e. 0

13. Gauss' Theorem states $\iiint\limits_{H} \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial H} \vec{F} \cdot d\vec{S}$



Compute either integral for the solid hemisphere, H,

given by $x^2 + y^2 + z^2 \le 4$ with $z \ge 0$

and the vector field $\vec{F} = (xz^2, yz^2, 0)$.

Notice that the boundary of the solid hemisphere ∂H consists of the hemisphere surface S given by $x^2 + y^2 + z^2 = 4$ with $z \ge 0$ and the disk D given by $x^2 + y^2 \le 4$ with $z = 0$.

a. $\dfrac{64\pi}{15}$

b. $\dfrac{128\pi}{15}$

c. $\dfrac{8}{3}\pi^2$

d. $\dfrac{32}{3}\pi^2$

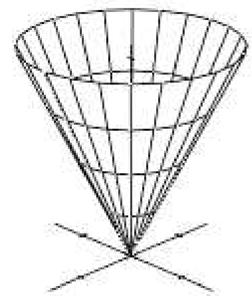
e. $\dfrac{64}{3}\pi^2$

Work Out: (Part credit possible. Show all work.)

14. (25 points) Verify Stokes' Theorem $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$

for the cone C given by $z = 2\sqrt{x^2 + y^2}$ for $z \le 8$

oriented up and in, and the vector field $\vec{F} = (yz, -xz, z)$.



Be sure to check and explain the orientations. Use the following steps:

a. Note: The cone may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r)$

Compute the surface integral by successively finding:

$\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

Recall: $\vec{F} = (yz, -xz, z)$

b. Compute the line integral by parametrizing the boundary curve and successively finding:

$$\vec{r}(\theta), \quad \vec{v}, \quad \vec{F}(\vec{r}(\theta)), \quad \oint_{\partial C} \vec{F} \cdot d\vec{s}$$

15. (15 points) A rectangular solid sits on the xy-plane with its top four vertices on the paraboloid $z = 4 - x^2 - 4y^2$. Find the dimensions and volume of the largest such box.