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MATH 251/253 Quiz 2 Spring 2008

Section 508/200,501,502 Solutions P. Yasskin

Multiple Choice: (5 points each)

**1.** A triangle has vertices $P = (-1,2,-3)$, $Q=(3,2,1)$, and $R = (-1,-1,0)$.

Find a vector perpendicular to the plane of the triangle.

**a.** $(1,-1,-1)$ Correct Choice

**b.** $(1,1,-1)$

**c.** $(-1,-1,-1)$

**d.** $(1,-1,1)$

**e.** $(15,-8,-12)$

$\overrightarrow{PQ} = Q - P = (4,0,4) \qquad \overrightarrow{PR} = R - P = (0,-3,3)$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 0 & 4 \\ 0 & -3 & 3 \end{vmatrix} = \hat{\imath}(0 - -12) - \hat{\jmath}(12 - 0) + \hat{k}(-12 - 0) = (12,-12,-12)$$

This is parallel to $(1,-1,-1)$.

**2.** A triangle has vertices $P = (-1,2,-3)$, $Q=(3,2,1)$, and $R = (-1,-1,0)$. Find its area.

**a.** $3$

**b.** $18$

**c.** $36$

**d.** $6\sqrt{3}$ Correct Choice

**e.** $12\sqrt{3}$

$\overrightarrow{PQ} \times \overrightarrow{PR} = (12,-12,-12)$ as in #1.

$$A = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{1}{2}\sqrt{144+144+144} = 6\sqrt{3}$$

**3**. Find an equation of the plane containing the triangle with vertices

$P = (-1,2,-3)$, $Q=(3,2,1)$, and $R = (-1,-1,0)$.

**a**. $x - y - z = 1$

**b**. $x + y - z = 1$

**c**. $x - y - z = 0$ Correct Choice

**d**. $x + y - z = 0$

**e**. $x + y - z = -1$

$\overrightarrow{PQ} \times \overrightarrow{PR} = (12,-12,-12)$ as in #1.

$\vec{N} \cdot X = \vec{N} \cdot P \qquad 12x - 12y - 12z = 12(-1) - 12(2) - 12(-3) = 0$ or $x - y - z = 0$

**4**. Find the point where the line $(x,y,z) = (3,2,1) + t(1,2,3)$ intersects the plane $x - y + z = -2$.

**a**. $(5,14,7)$

**b**. $(5,6,7)$

**c**. $(1,2,-1)$

**d**. $(1,2,-5)$

**e**. $(1,-2,-5)$ Correct Choice

$x = 3 + t, \quad y = 2 + 2t, \quad z = 1 + 3t \qquad x + y - z = (3 + t) - (2 + 2t) + (1 + 3t) = 2 + 2t = -2$

$t = -2 \qquad (x,y,z) = (3,2,1) - 2(1,2,3) = (1,-2,-5)$

**5**. (10 points) Consider the quadratic equation $x^2 - y^2 + 4z^2 + 2x - 6y - 8z = 8$

**a**. Complete the squares and bring the equation into standard form.

$(x^2 + 2x + 1) - (y^2 + 6y + 9) + 4(z^2 - 2z + 1) = 8 + 1 - 9 + 4 = 4$

$$\frac{(x+1)^2}{4} - \frac{(y+3)^2}{4} + (z-1)^2 = 1$$

**b**. Identify the equation as one of the following and find the indicated quantities:

- sphere: center, radius
- ellipsoid: center, radii
- hyperboloid: center, axis ($x$, $y$ or $z$), 1-sheet or 2-sheets, asymptotic cone
- cone: vertex, axis ($x$, $y$ or $z$)
- elliptic paraboloid: vertex, direction it opens ($+x$, $-x$, $+y$, $-y$, $+z$ or $-z$)
- hyperbolic paraboloid: vertex, axis ($x$, $y$ or $z$)
- cylinder: type (circular, elliptic, hyperbolic, parabolic), axis ($x$, $y$ or $z$)

This is a hyperboloid. The center is $(-1,-3,1)$. The axis is parallel to the $y$-axis.

Rewriting as $\frac{(x+1)^2}{4} + (z-1)^2 = 1 + \frac{(y+3)^2}{4}$ we see $\frac{(x+1)^2}{4} + (z-1)^2 \geq 1$.

So there is 1 sheet. The asymptotic cone is $\frac{(x+1)^2}{4} - \frac{(y+3)^2}{4} + (z-1)^2 = 0$.