Name________________________

MATH 251 Exam 1 Version H Fall 2018

Sections 200/202 P. Yasskin

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|---|---|---|---|
| 1-9 | /54 | 11 | /16 |
| 10 | /36 | Total | /106 |

Multiple Choice: (6 points each. No part credit.)

**1**. The Galactic Federation is trying to keep a stasis pod stationary in intergalactic space where there is no gravity. They already have $2$ tractor beams pulling on the pod with the forces

$$\vec{F}_1 = \langle 4,1,-3 \rangle \quad \text{and} \quad \vec{F}_2 = \langle -2,2,1 \rangle$$

If they apply a $3^{\text{rd}}$ tractor beam on the pod, what should its force $\vec{F}_3$ be to keep the pod stationary?

**a**. $\vec{F}_3 = \langle 2,3,-2 \rangle$

**b**. $\vec{F}_3 = \langle 2,-3,-2 \rangle$

**c**. $\vec{F}_3 = \langle -2,3,2 \rangle$

**d**. $\vec{F}_3 = \langle -2,-3,2 \rangle$

**e**. $\vec{F}_3 = \langle 2,3,2 \rangle$

**2**. The Galactic Federation moves a stasis pod from $(2,3,4)$ to $(6,9,0)$ by applying the $2$ forces:

$$\vec{F}_1 = \langle 4,1,-3 \rangle \quad \text{and} \quad \vec{F}_2 = \langle -2,2,1 \rangle$$

How much work is done by the force $\vec{F}_1$ only?

**a**. $W = 34$

**b**. $W = 33$

**c**. $W = 22$

**d**. $W = 10$

**e**. $W = 0$

**3**. If a satelite travels from West to East with constant speed in a great circle directly above the Equator of the Earth, where does the unit binormal $\hat{B}$ point?

**a**. North

**b**. South

**c**. West

**d**. Up

**e**. Down

**4**. Convert the polar equation $r = \dfrac{\cos\theta}{\sin^2\theta}$ to rectangular coordinates and identify the shape of the curve.

**a**. Circle of radius $4$ centered at a point on the $x$-axis.

**b**. Circle of radius $4$ centered at a point on the $y$-axis.

**c**. Circle of radius $2$ centered at a point on the $x$-axis.

**d**. Circle of radius $2$ centered at a point on the $y$-axis.

**e**. Parabola opening to the right.

**5**. Find the angle between the direction of the line $(x,y,z) = (3+t, 3-t, 4)$ and the normal to the plane $2x - y + z = 7$.

**a**. $0°$

**b**. $30°$

**c**. $45°$

**d**. $60°$

**e**. $90°$

**6**. Find the point where the line $(x,y,z) = \vec{r}(t) = (t+2, t-2, 2t-1)$ intersects the plane $3x - y + 2z = 12$. At this point $x + y + z =$

**a**. $-3$

**b**. $-1$

**c**. $0$

**d**. $1$

**e**. $3$

7. Is the permutation $p = (2,4,5,6,1,3)$ odd or even and find its inverse $\bar{p}$.

a. Odd $\bar{p} = (3,1,6,5,4,2)$

b. Odd $\bar{p} = (4,3,2,6,1,5)$

c. Odd $\bar{p} = (5,1,6,2,3,4)$

d. Even $\bar{p} = (4,3,2,6,1,5)$

e. Even $\bar{p} = (5,1,6,2,3,4)$

8. Find the equation of the hyperplane in $\mathbb{R}^4$ thru the point $P = (1,2,3,5)$ tangent to the vectors $\vec{a} = \langle 1,0,1,0\rangle$, $\vec{b} = \langle 0,1,0,1\rangle$ and $\vec{c} = \langle 1,1,0,0\rangle$. Let the general point be $X = (x,y,z,w)$. (Show your work. I may give part credit.)

a. $x - y - z + w = 1$

b. $x - y - z + w = -1$

c. $x + y - z - w = -4$

d. $x + y - z - w = 4$

e. $x + y + z + w = 11$

9. Find the volume of the parallepiped in $\mathbb{R}^4$ with adjacent edges $\vec{a} = \langle 1,0,1,0\rangle$, $\vec{b} = \langle 0,1,0,1\rangle$ and $\vec{c} = \langle 1,1,0,0\rangle$. (Show your work. I may give part credit.)

a. $1$

b. $\sqrt{2}$

c. $2$

d. $2\sqrt{2}$

e. $4$

Work Out: (Points indicated. Part credit possible. Show all work.)

**10**. (36 points) For the twisted cubic $\vec{r}(t) = \left(\frac{t^3}{3}, t^2, 2t\right)$ compute each of the following:

**a**. (6 pts) The velocity $\vec{v}$

$\vec{v} =$ ____________________

**b**. (6 pts) The speed $\frac{ds}{dt}$ (Simplify!)

$\frac{ds}{dt} =$ ____________________

**c**. (6 pts) The tangential acceleration $a_T$

$a_T =$ ____________________

**d**. (6 pts) The mass of a wire in the shape of this twisted cubic between $(0,0,0)$ and $(9,9,6)$ if the linear mass density is $\delta = yz$.

$M =$ ____________________

**e**. (6 pts) The $y$-component of the center of mass of the wire between $(0,0,0)$ and $(9,9,6)$ if the linear mass density is $\delta = yz$.

$\bar{y} =$ ____________________

**f**. (6 pts) The work done to move a bead along of a wire in the shape of this twisted cubic between $(0,0,0)$ and $(9,9,6)$ by the force $\vec{F} = (z, 2y, -3x)$.

$W =$ ____________________

**11**. (15 points) Write the vector $\vec{a} = \langle 2,2,6 \rangle$ as the sum of two vectors $\vec{b}$ and $\vec{c}$ with $\vec{b}$ parallel to $\vec{d} = \langle 1,-1,2 \rangle$ and $\vec{c}$ perpendicular to $\vec{d}$.
Check $\vec{c}$ is perpendicular to $\vec{d}$.

$\vec{a} =$ ____________________ + ____________________

$\vec{b}$ $\vec{c}$