Name______________________

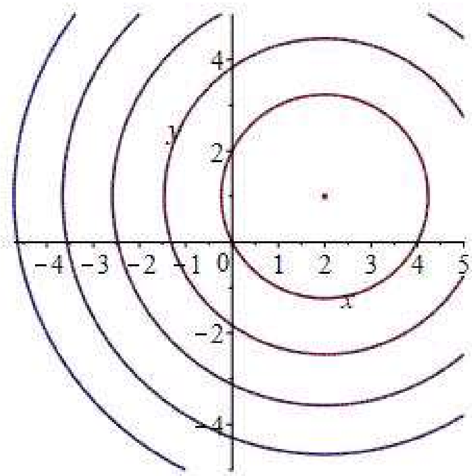
1-11	/55	13	/25
12	/20	EC	/5
		Total	/105

MATH 251 Exam 2 Version B Fall 2018

Sections 504/505 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Which of these functions has the contour plot at the right?



a. $\sqrt{x^2+y^2-4x-2y+9}$

b. $\sqrt{x^2+y^2+4x+2y+5}$

c. $\sqrt{x^2+y^2+4x-2y+1}$

d. $x^2+y^2-4x+2y$

e. $x^2+y^2+4x+2y$

2. If $f = x\cos y - y\sin x$ which of the following is INCORRECT?

a. $\dfrac{\partial^3 f}{\partial x \partial x \partial x} = y\cos x$

b. $\dfrac{\partial^3 f}{\partial y \partial x \partial x} = \sin x$

c. $\dfrac{\partial^3 f}{\partial x \partial y \partial x} = -\sin x$

d. $\dfrac{\partial^3 f}{\partial x \partial x \partial y} = \sin x$

e. $\dfrac{\partial^3 f}{\partial y \partial y \partial y} = x\sin y$

3. The partial derivative $\left.\frac{\partial f}{\partial x}\right|_{(2,3)}$ gives the

a. slope at $y = 3$ of the x-trace of f with x fixed at 2.

b. slope at $x = 2$ of the x-trace of f with y fixed at 3.

c. slope at $y = 3$ of the y-trace of f with x fixed at 2.

d. slope at $x = 2$ of the y-trace of f with y fixed at 3.

4. Find the tangent plane to the graph of $z = x^2 y^3$ at $(x,y) = (2,1)$. The z-intercept is

a. 20

b. 16

c. 4

d. -16

e. -20

5. The equation $x^3 z^3 - y^2 z^2 = -1$ implicitly defines z as a function of x and y.
Find $\frac{\partial z}{\partial y}$ at $(x,y,z) = (2,3,1)$.

a. 2

b. 1

c. 0

d. -1

e. -2

6. Find the equation of the plane tangent to the surface $x^3 z^3 - y^2 z^2 = -1$ at $(x,y,z) = (2,3,1)$.
The z-intercept is

a. $c = -12$

b. $c = -2$

c. $c = 2$

d. $c = 6$

e. $c = 12$

7. The strength, S, of a support beam of length L, width W and height H is given by $S = \frac{WH^2}{L}$. Currently, $L = 50$ cm, $W = 5$ cm and $H = 10$ cm. Use the linear approximation to estimate the change in the strength if L increases by 5 cm, W increases by 0.5 cm and H increases by 1 cm?

a. 10

b. 8

c. 6

d. 4

e. 2

8. Dark Invader is flying through a dark matter field whose density is given by $\delta = xyz^2$. If Dark's current position is $\vec{r}(2) = \langle 3,2,1\rangle$ and his velocity is $\vec{v}(2) = \langle 1,2,1\rangle$, find the rate at which the density of dark matter is changing as seen by Dark.

a. 10

b. $10\sqrt{6}$

c. $20\sqrt{6}$

d. 20

e. $\frac{20}{\sqrt{6}}$

9. When there is no wind, a weather balloon floats in the direction of **decreasing** air density. If the air density is $\delta = x^2 + y^2 + z^3$ and the balloon is located at $(x,y,z) = (2,6,1)$, find the vector direction in which the balloon floats.

a. $\left\langle \frac{4}{13}, \frac{12}{13}, \frac{3}{13} \right\rangle$

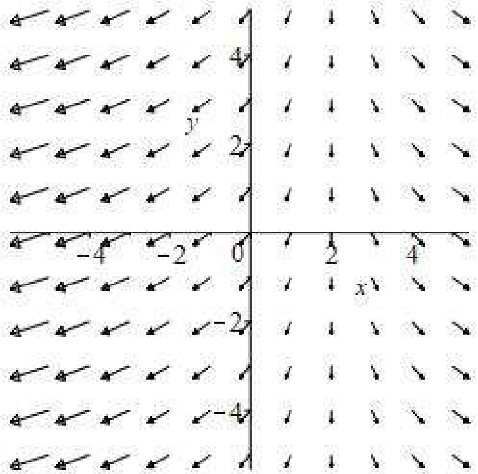
b. $\left\langle \frac{-4}{13}, \frac{12}{13}, \frac{-3}{13} \right\rangle$

c. $\left\langle \frac{-4}{13}, \frac{-12}{13}, \frac{-3}{13} \right\rangle$

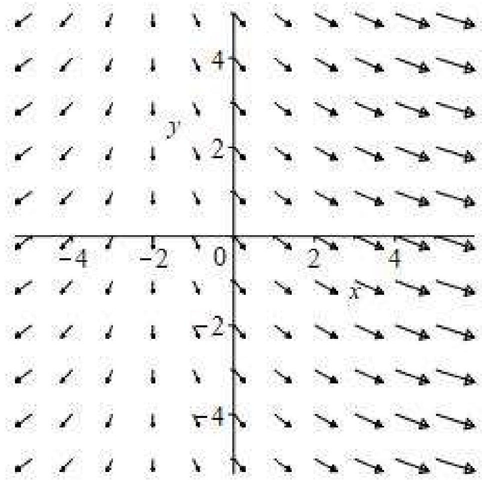
d. $\left\langle \frac{4}{13}, \frac{-12}{13}, \frac{3}{13} \right\rangle$

10. Which is the plot of the vector field $\vec{F} = \langle x-2, -2\rangle$?

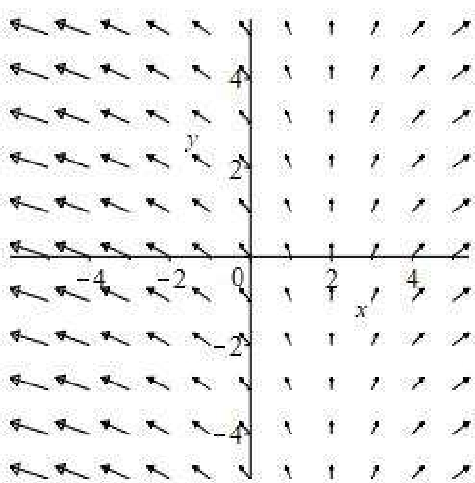
a.



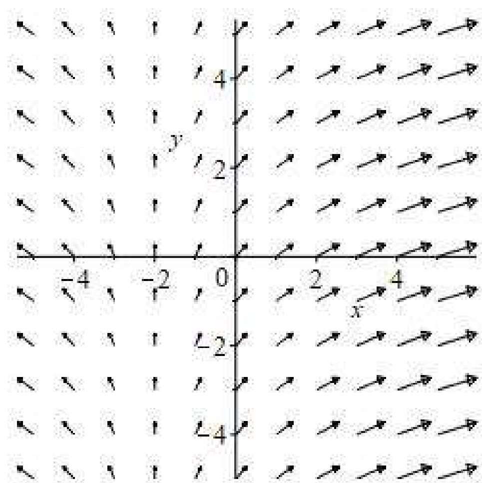
c.



b.



d.



11. Find a scalar potential, $f(x,y,z)$, for $\vec{F} = \left\langle -\frac{yz}{x^2}, \frac{z}{x}, \frac{y}{x} \right\rangle$. Then $f(4,4,4) - f(1,1,1) =$

a. 1

b. 2

c. 3

d. 4

e. 5

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (20 points) Find the point(s), $X = (x, y, z)$, on the hyperboloid $x^2 + y^2 - z^2 = 1$ where the normal vector points in the same direction as $\vec{v} = \langle 1, 3, -3 \rangle$.

13. (25 points+5 points extra credit) Find the point, $X = (x, y, z)$, on the upper half of the hyperboloid $x^2 + y^2 - z^2 = 1$ which is closest to the point $P = (6, 8, 0)$. What is the distance?

You may solve by either method. There is 5 points extra credit for solving by both methods.

Method: **Lagrange Multipliers**::

Method: **Eliminate the Constraint**: