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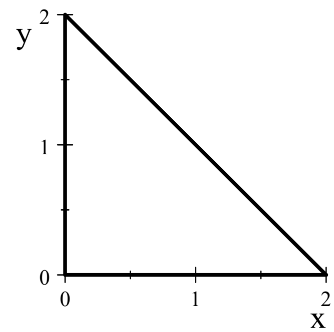
MATH 251 Final Version A Fall 2018

Sections 504 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1-10	/60	12	/20
11	/ 5	13	/25
		Total	/110

1. Compute $I = \int_{\partial R} (2y + 3x^2y^2)\,dx + (5x + 2x^3y)\,dy$ over the complete boundary of the triangle shown at the right traversed counterclockwise.
HINT: Use a theorem.



a. $I = 3$

b. $I = 6$

c. $I = 7$

d. $I = 12$

e. $I = 14$

2. Compute $\int_{(1,1,1)}^{(8,4,2)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = \langle y^2z^2, 2xyz^2, 2xy^2z \rangle$ along the curve $\vec{r}(t) = \langle t^3, t^2, t \rangle$.
HINT: Find a scalar potential.

a. -512

b. -511

c. 0

d. 511

e. 512

3. Compute $\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the quartic surface $z = x^4 + 2x^2y^2 + y^4$ with $z \le 16$ oriented **up** and **in**, for $\vec{F} = \langle -yz, xz, z^2 \rangle$.
HINT: Use a theorem.

a. 128π

b. 256π

c. 512π

d. 1024π

e. 2048π

4. The two legs of a right triangle are $\vec{a}$ and $\vec{b}$ and the hypotenuse is $\vec{c}$. So $\vec{a} \perp \vec{b}$ and $\vec{c} = \vec{a} + \vec{b}$. Given that $\vec{c} = \langle 9, 9, -9 \rangle$ and the direction of $\vec{a}$ is $\hat{a} = \left\langle \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3} \right\rangle$, find the magnitude $\vec{b}$.

a. $\left|\vec{b}\right| = 36$

b. $\left|\vec{b}\right| = 18\sqrt{2}$

c. $\left|\vec{b}\right| = 18$

d. $\left|\vec{b}\right| = 9\sqrt{2}$

e. $\left|\vec{b}\right| = 9$

5. An ant is walking across a frying pan where the temperature is $T = \frac{1}{12}x^3y^2$. If the ant is currently at $P = (2, 3)$, in what unit vector direction should the ant walk to reduce the temperature as fast as possible?

a. $\left\langle \frac{9}{13}, \frac{4}{13} \right\rangle$

b. $\left\langle \frac{-9}{5}, \frac{-4}{5} \right\rangle$

c. $\left\langle \frac{9}{5}, \frac{4}{5} \right\rangle$

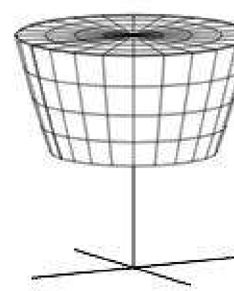
d. $\left\langle \frac{9}{\sqrt{97}}, \frac{4}{\sqrt{97}} \right\rangle$

e. $\left\langle \frac{-9}{\sqrt{97}}, \frac{-4}{\sqrt{97}} \right\rangle$

6. The point $(1,2)$ is a critical point of the function $f(x,y) = 16x^4 + y^4 - 32xy$.
Classify the point $(1,2)$ using the Second Derivative Test.

a. Local Mininum
b. Local Maximum
c. Saddle Point
d. Inflection Point
e. Test Fails

7. Find the mass of the piece of the solid paraboloid $z = x^2 + y^2$ for $2 \le z \le 4$ if the density is $\delta = z$.



a. 64π
b. 60π
c. $\frac{112}{3}\pi$
d. $\frac{56}{3}\pi$
e. 20π

8. Find the center of mass of the piece of the solid paraboloid $z = x^2 + y^2$
for $2 \le z \le 4$ if the density is $\delta = z$.

a. $\frac{14}{75}$
b. $\frac{14}{15}$
c. $\frac{45}{14}$
d. $\frac{14}{45}$
e. $\frac{15}{14}$

9. Find the equation of the plane tangent to the hyperboloid $xyz = 6$ at the point $(3,2,1)$.

a. $(x,y,z) = (3+2t, 2+3t, 1+6t)$

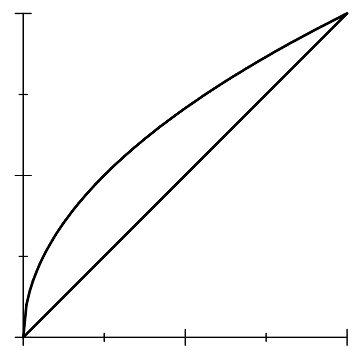
b. $(x,y,z) = (2+3t, 3+2t, 6+t)$

c. $3x + 2y + z = 14$

d. $3x + 2y + z = 18$

e. $2x + 3y + 6z = 18$

10. Find the volume under the surface $z = 2xy$ above the region bounded by $y = x$ and $y = 2\sqrt{x}$. The base is shown at the right.



a. $\frac{128}{3}$

b. $\frac{128}{5}$

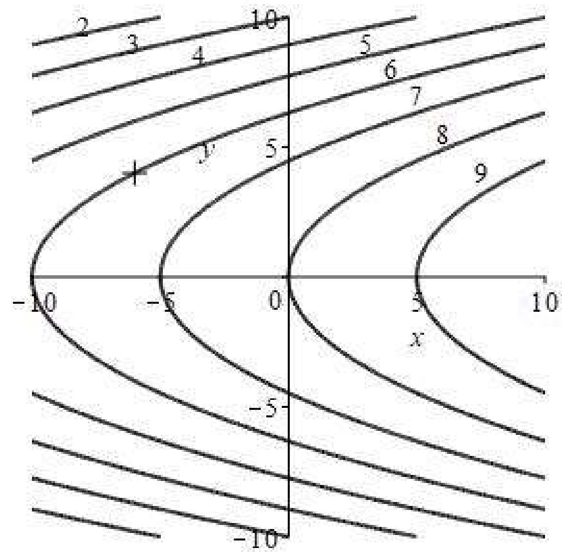
c. $\frac{64}{3}$

d. $\frac{64}{5}$

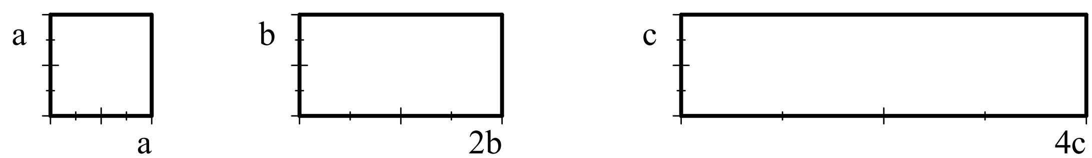
e. $\frac{64}{7}$

Work Out: (Points indicated. Part credit possible. Show all work.)

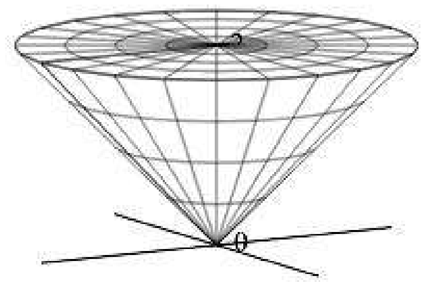
11. (5 points) At the right is the contour plot of a function $f(x,y)$. The contours are labeled by the function values. If you start at the cross at $(-6,4)$ and move so that your velocity is always in the direction of $\vec{\nabla} f$, the gradient of f, roughly sketch your path on the plot.



12. (20 points) A 118 cm wire is cut into 3 pieces. As shown in the plots, one piece is bent into a square of side a. Another piece is bent into a rectangle with sides b and $2b$. The third piece is bent into a rectangle with sides c and $4c$. Note: a, b and c may not be the same length, even if they look that way in the plots. Find a, b and c which minimize the total area enclosed in the three shapes. Note: the constraint is the sum of the perimeters.
You do NOT need to check it is a minimum rather than a maximum.



13. (25 points) Verify Gauss' Theorem $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = \langle -xz^2, -yz^2, z^3 \rangle$ and the solid above the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 2$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence and give the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $dV =$

b. Compute the left hand side:

$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of the cone C and a disk D with appropriate orientations.

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c. Parametrize the disk D:

$\vec{R}(r,\theta) =$

d. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

e. Compute the normal vector:

$\vec{N} =$

f. Evaluate $\vec{F} = \langle -xz^2, -yz^2, z^3 \rangle$ on the disk:

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

g. Compute the dot product:

$\vec{F} \bullet \vec{N} =$

h. Compute the flux through D:

$\iint_D \vec{F} \bullet d\vec{S} =$

Parametrize the cone C as $\vec{R}(r,\theta) = \langle r\cos\theta, r\sin\theta, r \rangle$

i. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

j. Compute the normal vector:

$\vec{N} =$

k. Evaluate $\vec{F} == \langle -xz^2, -yz^2, z^3 \rangle$ on the cone:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

l. Compute the dot product

$\vec{F} \bullet \vec{N} =$

m. Compute the flux through C:

$\iint_C \vec{F} \bullet d\vec{S} =$

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n. Compute the **TOTAL** right hand side:

$\iint_{\partial V} \vec{F} \bullet d\vec{S} =$