Name_____

MATH 251 Exam 1 Version B Fall 2020

Sections 519 P. Yasskin

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|---|---|---|---|
| 1-9 | /45 | 12 | /15 |
| 10 | /5 | 13 | /28 |
| 11 | /10 | Total | /103 |

Multiple Choice: (5 points each. No part credit.)

**1**. The points $A = (3,2,1)$ and $B = (5,6,5)$ are the endpoints of the diameter of a sphere. If $C = (a,b,c)$ is the center and $r$ is the radius, what is $a+b+c+r$?

**a**. $40$

**b**. $31$

**c**. $29$

**d**. $20$

**e**. $14$

**2**. If $\vec{u}$ points NorthWest and $\vec{v}$ points Down, in what direction does $\vec{u}\times\vec{v}$ point?

**a**. SouthEast

**b**. SouthWest

**c**. NorthWest

**d**. Up

**3**. The Galactic Federation is trying to keep a stasis pod stationary in intergalactic space where there is no gravity. They already have $2$ tractor beams pulling on the pod with the forces

$$\vec{F}_1 = \langle -3,2,1\rangle \qquad \vec{F}_2 = \langle -1,-2,3\rangle$$

They now apply a $3^{\text{rd}}$ tractor beam with the force, $\vec{F}_3 = \langle a,b,c\rangle$, to keep the pod stationary. What is $a+b+c$?

**a**. $-8$

**b**. $-2$

**c**. $0$

**d**. $2$

**e**. $8$

**4**. The thrusters on the Starship Galileo exert the force $\vec{F} = \langle 4, -2, 1 \rangle$ which moves the ship from $P = (1, 5, 2)$ to $Q = (3, 4, 5)$. Find the work done by the thrusters.

**a**. $W = 15$

**b**. $W = 13$

**c**. $W = 11$

**d**. $W = 9$

**e**. $W = 7$

**5**. Find the tangent vector, $\vec{v}$, to the curve $\vec{r}(t) = (t^3, t^2, t)$ at the point $(8, 4, 2)$. Then find it dot product with $\vec{F} = \langle 3, 2, 1 \rangle$.

**a**. $\vec{F} \cdot \vec{v} = (12, 4, 1)$

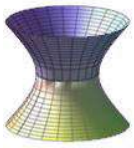
**b**. $\vec{F} \cdot \vec{v} = (36, 8, 1)$

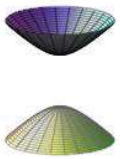
**c**. $\vec{F} \cdot \vec{v} = 19$

**d**. $\vec{F} \cdot \vec{v} = 29$

**e**. $\vec{F} \cdot \vec{v} = 45$

**6**. Which of the following is the graph of the equation $(x-1)^2 + (y-3)^2 - z = 4$?

a 

b 

c 

d 

**7**. A point has spherical coordinates $(\rho,\phi,\theta) = \left(\sqrt{2}, \frac{\pi}{6}, \frac{\pi}{4}\right)$. If its rectangular coordinates are $(x,y,z)$, then $xyz =$

**a**. $\frac{\sqrt{6}}{8}$

**b**. $\frac{\sqrt{6}}{4}$

**c**. $\frac{\sqrt{3}}{4}$

**d**. $\frac{3\sqrt{2}}{8}$

**e**. $\frac{3\sqrt{2}}{4}$

**8**. Find the area of the triangle with two edges $\vec{v} = \langle 4,0,1\rangle$ and $\vec{w} = \langle 2,1,-1\rangle$.

**a**. $A = \frac{1}{2}\sqrt{53}$

**b**. $A = \sqrt{53}$

**c**. $A = \frac{1}{2}\sqrt{54}$

**d**. $A = \sqrt{54}$

**e**. $A = 27$

**9**. Find the volume of the parallelepiped with edges $\vec{u} = \langle 2,-3,1\rangle$, $\vec{v} = \langle 4,0,1\rangle$ and $\vec{w} = \langle 2,1,-1\rangle$.

**a**. $V = -20$

**b**. $V = -16$

**c**. $V = 8$

**d**. $V = 16$

**e**. $V = 20$

Work Out: (Points indicated. Part credit possible. Show all work.)

**10**. (5 points) Find a parametric equation of the line which is perpendicular to the plane $3x + 2y - z = 4$ and passes through the point $(3, 5, 1)$.

**11**. (10 points) Find a normal equation of the plane which contains the line $(x, y, z) = (3 - 2t, 2 + t, 2 + 2t)$ and passes through the point $(3, 4, 1)$.

**12**. (15 points) Consider the two planes

$$y + z = 3$$
$$2x + 2y + z = 4$$

**a**. (4 pts) Find the angle (in degrees) between the planes.

**b**. (4 pts) Find a direction vector, $\vec{v}$, for the line of intersection of the planes.

**c**. (4 pts) Find a point, $P$, on the line of intersection of the planes.

**d**. (3 pts) Find a parametric equation for the line of intersection of the planes.

**13**. (28 points) For the parametric curve $\vec{r}(t) = \left(t^2, \frac{1}{3}t^3, 2t\right)$ compute each of the following:

**a**. (3 pts) velocity $\vec{v}$

$\vec{v} =$ _____

**b**. (3 pts) acceleration $\vec{a}$

$\vec{a} =$ _____

**c**. (3 pts) jerk $\vec{j}$

$\vec{j} =$ _____

**d**. (2 pts) speed $|\vec{v}|$ (Simplify!)
HINT: The quantity inside the square root is a perfect square.

$|\vec{v}| =$ _____

**e**. (2 pts) tangential acceleration $a_T$

$a_T =$ _____

**f**. (2 pts) the values of $t$ where the curve passes thru the points

$A = \left(1, \frac{1}{3}, 2\right)$ $t =$ _____

$B = (9,9,6)$ $t =$ _____

**g**. (4 pts) arc length between $\left(1, \frac{1}{3}, 2\right)$ and $(9,9,6)$

$L =$ _____

**h**. (4 pts) A wire has the shape of this curve between $\left(1, \frac{1}{3}, 2\right)$ and $(9,9,6)$. Find the mass of the wire if the linear mass density is $\delta = xz$.

$M =$ _____

**i**. (4 pts) A wire has the shape of this curve. Find the work done by the force $\vec{F} = (0,z,y)$ which pushes a bead along the wire from $\left(1, \frac{1}{3}, 2\right)$ to $(9,9,6)$.

$W =$ _____