Name________________________

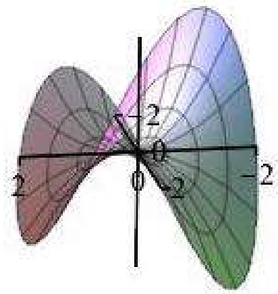
MATH 251 Exam 2 Version A Fall 2020

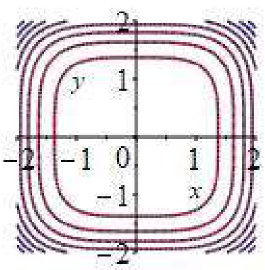
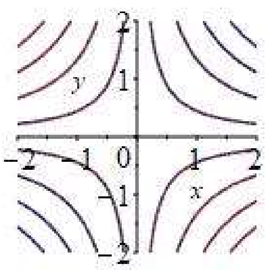
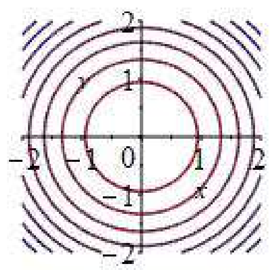
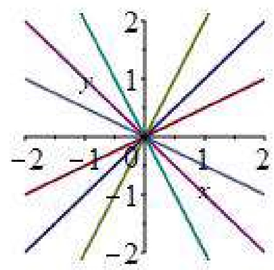
Sections 517 P. Yasskin

1-10	/50	12	/18
11	/12	13	/20+10EC
		Total	/100+10EC

Multiple Choice: (5 points each. No part credit.)

1. Which of the following is the contour plot for the function whose graph is shown at the right?

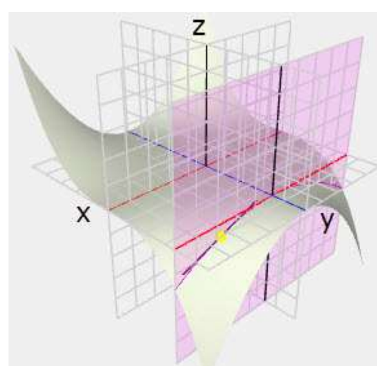


a  b  c  d 

2. Identify the domain and image of the function $w = f(x, y, z) = \ln(9 - x^2 - y^2 - z^2)$.
Be sure to indicate whether the boundary of any region is included or not.
Enter each answer as an inequality or an interval.

3. Find the tangent plane to the graph of $z = x^2 y^3$ at the point $(x, y) = (2, 1)$. Then find its z-intercept.

4. In the figure at the right, the vertical plane intersects the surface in a curve which is either the x-Trace or the y-Trace. Which is it? The slope of the tangent line to this trace is either the x-Partial Derivative or the y-Partial Derivative. Which is it?



5. Suppose $w = w(x,y,z)$ while $x = s^2$, $y = t^3$ and $z = s^3 + t^2$, find $\left.\frac{\partial w}{\partial t}\right|_{(1,1)}$ given that

$$\left.\frac{\partial w}{\partial x}\right|_{(1,1,2)} = 3 \qquad \left.\frac{\partial w}{\partial y}\right|_{(1,1,2)} = 4 \qquad \left.\frac{\partial w}{\partial z}\right|_{(1,1,2)} = 5$$

6. The equation $x^2z^3 + y^3z^2 = 17$ defines a surface which passes thru the point $(3,2,1)$. This surface implicitly defines a function $z = f(x,y)$ passing through this point. Find $\frac{\partial f}{\partial y}(3,2)$.

7. Find the equation of the tangent plane to the hyperboloid $3(x-3)^2 - (y-2)^2 - (z-4)^2 = 1$ at the point $(x,y,z) = (2,3,3)$.

8. A cardboard box has length $L = 5\,cm$, width $W = 4\,cm$ and height $H = 3\,cm$. Use the linear approximation to estimate the volume of cardboard used to make this box if the thickness of cardboard on the bottom is $0.8\,cm$, the thickness of cardboard on the 4 sides is $0.4\,cm$, and the thickness of cardboard on the top is $0.2\,cm$.
Note: There are 2 thicknesses of cardboard in each direction.

9. If 2 resistors with resistances R_1 and R_2 are arranged in parallel, then the net resistance R is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Initially, $R_1 = 2\Omega$, $R_2 = 4\Omega$. First find the initial value of R. If R_1 is increasing at $\frac{dR_1}{dt} = 0.3\,\frac{\Omega}{\sec}$ while R_2 is decreasing at $\frac{dR_2}{dt} = -0.3\,\frac{\Omega}{\sec}$, find the rate at which the net resistance is changing $\frac{dR}{dt}$. Be careful to get the sign correct.

Enter exact numbers. For example, $\frac{-0.5}{7}$ should be entered as -0.5/7, NO SPACES, NO UNITS.

10. The point $(-4,1)$ is a critical point of the function $g(x,y) = x^2y - 2y^3x + 10xy$. Apply the 2nd-Derivative Test to classify $(-4,1)$.

- **a**. Local Minimum
- **b**. Local Maximum
- **c**. Inflection Point
- **d**. Saddle Point
- **e**. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (12 points) Find all 1^{st} and 2^{nd} partial derivatives of $f(x,y) = x^2 \sin(xy)$.
Enter f_{yy} here but put all of them on your paper.

12. (18 points) The ideal gas law says the pressure, P, the density, δ, and the temperature, T, are related by $P = k\delta T$ where k is a constant. A weather balloon measures that at its current position,

$$P = .81\,atm \qquad \delta = 1.2\,\frac{kg}{m^3} \qquad T = 270°K$$

The weather balloon also measures that gradients of the density and temperature are

$$\vec{\nabla}\delta = \left\langle \frac{\partial\delta}{\partial x}, \frac{\partial\delta}{\partial y}, \frac{\partial\delta}{\partial z} \right\rangle = \langle .2, .1, -.2\rangle \frac{kg/m^3}{m}$$

$$\vec{\nabla}T = \left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right\rangle = \langle 3, -12, 4\rangle \frac{°K}{m}$$

a. (2 pts) Find the constant k.

b. (9 pts) Find the gradient of the pressure.
HINT: Find each component separately. No need to simplify numbers.
Enter $\dfrac{\partial P}{\partial x}$ here but put all three components on your paper.

c. (4 pts) In the absence of wind or other forces, a balloon will tend to drift from regions of high density to regions of low density. In what unit vector direction, $\hat{u}$, will this balloon drift?

d. (3 pts) If the balloon's current velocity is $\vec{v} = \langle 4, 1, 3\rangle \frac{m}{\sec}$, find the rate the temperature is changing as seen by the balloon.

13. (20 points + 10 points extra credit) A rectangular solid box has 3 faces in the coordinate planes and the remaining vertex on the plane $z = 36 - 3x - 4y$. Find the dimensions and volume of the largest such box.
NOTE: Solve by either Eliminating a Variable or by Lagrange Multipliers. Extra Credit for solving by both methods. Draw a line across your paper to clearly separate the two solutions.

Solution by Eliminating a Variable:

Solution by Lagrange Multipliers: