Name________________________

MATH 251 Exam 2 Version B Fall 2020

Sections 519 P. Yasskin

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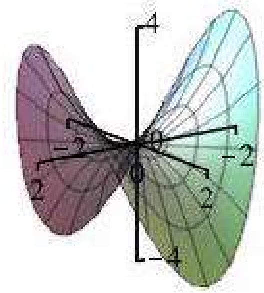
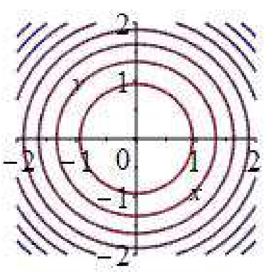
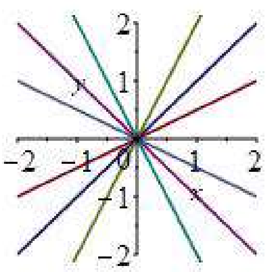
|---|---|---|---|
| 1-10 | /50 | 12 | /18 |
| 11 | /12 | 13 | /20+10EC |
| | | Total | /100+10EC |

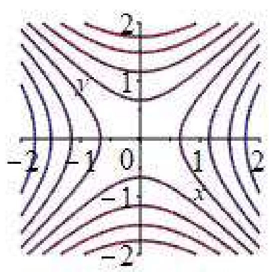
Multiple Choice: (5 points each. No part credit.)

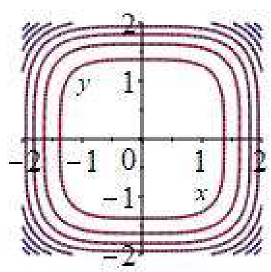
**1**. Which of the following is the contour plot for the function whose graph is shown at the right?



a 


b 


c 

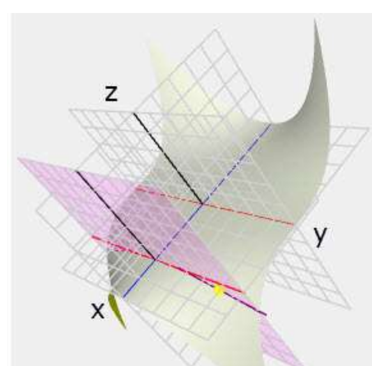

d 


**2**. Identify the domain and image of the function $w = f(x,y,z) = \dfrac{2}{\sqrt{4 - x^2 - y^2 - z^2}}$.

Be sure to say whether the boundary of any region is included or not.
Enter each answer as an inequality or an interval.

**3**. Find the tangent plane to the graph of $z = x^3y^2$ at the point $(x,y) = (1,2)$. Then find its $z$-intercept.

**4**. In the figure at the right, the vertical plane intersects the surface in a curve which is either the $x$-Trace or the $y$-Trace. Which is it? The slope of the tangent line to this trace is either the $x$-Partial Derivative or the $y$-Partial Derivative. Which is it?



**5**. Suppose $w = w(x,y,z)$ while $x = s^2$, $y = t^3$ and $z = s^3 + t^2$, find $\left.\frac{\partial w}{\partial s}\right|_{(1,1)}$ given that

$$\left.\frac{\partial w}{\partial x}\right|_{(1,1,2)} = 3 \qquad \left.\frac{\partial w}{\partial y}\right|_{(1,1,2)} = 4 \qquad \left.\frac{\partial w}{\partial z}\right|_{(1,1,2)} = 5$$

**6**. The equation $x^2z^3 + y^3z^2 = 17$ defines a surface which passes thru the point $(3,2,1)$. This surface implicitly defines a function $z = f(x,y)$ passing through this point. Find $\frac{\partial f}{\partial x}(3,2)$.

**7**. Find the equation of the tangent plane to the hyperboloid $(x-3)^2 - (y-2)^2 + (z-4)^2 = 1$ at the point $(x,y,z) = (2,3,3)$.

**8**. If $2$ resistors with resistances $R_1$ and $R_2$ are arranged in parallel, then the net resistance $R$ is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Initially, $R_1 = 2\Omega$, $R_2 = 4\Omega$. First find the initial value of $R$. If $R_1$ increases by $\Delta R_1 = 0.3\,\Omega$ while $R_2$ decreases by $\Delta R_2 = -0.3\,\Omega$, use the linear approximation to estimate the change in the net resistance $\Delta R$. Be careful to get the sign correct.

Enter exact numbers. For example, $\frac{-0.5}{7}$ should be entered as -0.5/7, NO SPACES, NO UNITS.

**9**. A cardboard box has length $L = 5\,cm$, width $W = 4\,cm$ and height $H = 3\,cm$. If the length is increasing at $\frac{dL}{dt} = 0.6\,\frac{cm}{\sec}$ and the width is decreasing at $\frac{dW}{dt} = -0.3\,\frac{cm}{\sec}$, while the **surface area** is held constant, find the rate that the height is changing, $\frac{dH}{dt}$. Be careful to get the sign correct.

HINT: What is the formula for the surface area?

**10**. The point $(-2,4)$ is a critical point of the function $g(x,y) = xy^3 - 16x^2y - 80xy$. Apply the 2nd-Derivative Test to classify $(-2,4)$.

- **a**. Local Minimum
- **b**. Local Maximum
- **c**. Inflection Point
- **d**. Saddle Point
- **e**. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

**11**. (12 points) Find all $1^{st}$ and $2^{nd}$ partial derivatives of $f(x,y) = y^2 \sin(xy)$.
Enter $f_{xx}$ here but put all of them on your paper.

**12**. (18 points) The ideal gas law says the pressure, $P$, the density, $\delta$, and the temperature, $T$, are related by $P = k\delta T$ where $k$ is a constant. A weather balloon measures that at its current position,

$$P = .81\,atm \qquad \delta = 1.5\,\frac{kg}{m^3} \qquad T = 270°K$$

The weather balloon also measures that gradients of the density and temperature are

$$\vec{\nabla}\delta = \left\langle \frac{\partial\delta}{\partial x}, \frac{\partial\delta}{\partial y}, \frac{\partial\delta}{\partial z} \right\rangle = \langle -.2, .2, -.1 \rangle \frac{kg/m^3}{m}$$

$$\vec{\nabla}T = \left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right\rangle = \langle -3, 4, -12 \rangle \frac{°K}{m}$$

**a**. (2 pts) Find the constant $k$.

**b**. (9 pts) Find the gradient of the pressure.
HINT: Find each component separately. No need to simplify numbers.
Enter $\dfrac{\partial P}{\partial y}$ here but put all three components on your paper.

**c**. (4 pts) In the absence of wind or other forces, a balloon will tend to drift from regions of high density to regions of low density. In what unit vector direction, $\hat{u}$, will this balloon drift?

**d**. (3 pts) If the balloon's current velocity is $\vec{v} = \langle 4, 1, 3 \rangle \frac{m}{\sec}$, find the rate the temperature is changing as seen by the balloon.

**13**. (20 points + 10 points extra credit) An aquarium has a marble base, a glass front and aluminum sides and back. There is no top. The marble costs $\$.50$ per $in^2$. The glass costs $\$.30$ per $in^2$. The aluminum costs $\$.10$ per $in^2$. Find the dimensions and cost of the aquarium with minimum cost if the volume needs to be $V = 5000\ in^3$. Let $x$ be the width of the front, left to right. Let $y$ be the width of each side, front to back. Let $z$ be the height, top to bottom.
HINT: Work in cents.
NOTE: Solve by either Eliminating a Variable or by Lagrange Multipliers. Extra Credit for solving by both methods. Draw a line across your paper to clearly separate the two solutions.

**Solution by Eliminating a Variable**:

**Solution by Lagrange Multipliers**: