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MATH 251 Exam 3 Version A Fall 2020

Sections 517 P. Yasskin

1-8	/48	10	/20
9	/20	11	/20
		Total	/108

Multiple Choice: (6 points each. No part credit.)

1. Compute $\int_0^2 \int_{x^2}^{2x} 2xy\,dy\,dx$.

a. $-\frac{16}{5}$

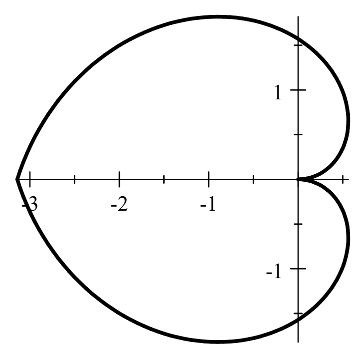
b. $\frac{32}{5}$

c. $\frac{16}{5}$

d. $\frac{32}{3}$

e. $\frac{16}{3}$

2. Find the area of the heart shaped region inside the polar curve $r = |\theta|$.
HINT: Double the upper half.



a. $\frac{\pi^3}{6}$

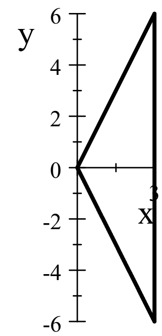
b. $\frac{\pi^3}{3}$

c. $\frac{4\pi^3}{3}$

d. $\frac{8\pi^3}{3}$

e. $\frac{16\pi^3}{3}$

3. Find the mass of a triangular plate with vertices $(0,0)$, $(3,6)$ and $(3,-6)$ whose surface mass density is $\delta = x$.



a. 12
b. 24
c. 36
d. 48
e. 60

4. Find the center of mass of a triangular plate with vertices $(0,0)$, $(3,6)$ and $(3,-6)$ whose surface mass density is $\delta = x$.

a. $\left(\frac{9}{4}, 0\right)$
b. $\left(0, \frac{9}{4}\right)$
c. $\left(\frac{4}{9}, 0\right)$
d. $\left(0, \frac{4}{9}\right)$
e. $(81, 0)$

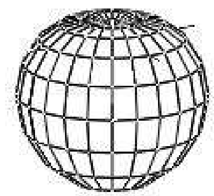
5. Compute $\iiint x^2 + y^2\, dV$ over the region between the cones $z = \sqrt{x^2 + y^2}$ and $z = 4 - \sqrt{x^2 + y^2}$.

a. $\frac{8\pi}{3}$
b. $\frac{16\pi}{3}$
c. $\frac{32\pi}{3}$
d. $\frac{16\pi}{5}$
e. $\frac{32\pi}{5}$

6. The surface of an apple is given in spherical coordinates by

$$\rho = 2 - 2\cos\varphi$$

Its volume is given by the integral:



a. $V = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{1-\cos\varphi} 1\, d\rho\, d\varphi\, d\theta$

b. $V = \int_0^{2\pi} \int_0^{\pi} \int_0^{2-2\cos\varphi} 1\, d\rho\, d\varphi\, d\theta$

c. $V = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{1-\cos\varphi} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

d. $V = \int_0^{2\pi} \int_0^{\pi} \int_0^{2-2\cos\varphi} \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

e. $V = \int_0^{2\pi} \int_0^{\pi} \int_0^{1} (2 - 2\cos\varphi)\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

7. If $f = x^2 + y^2 - 2z^2$ and $\vec{F} = (xz, yz, -z^2)$, which of the following is FALSE?

a. $\vec{\nabla} \times \vec{\nabla} f = \vec{0}$

b. $\vec{\nabla} \cdot \vec{\nabla} f = 0$

c. $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} = 0$

d. $\vec{\nabla}\left(\vec{\nabla} \cdot \vec{F}\right) = \vec{0}$

e. None of the above. They are all true.

8. Let f be a scalar potential for $\vec{F} = (y, x, z)$. Find $f(2,2,2) - f(0,0,0)$

a. 2
b. 4
c. 6
d. 8
e. 10

Work Out: (Points indicated. Part credit possible. Show all work.)

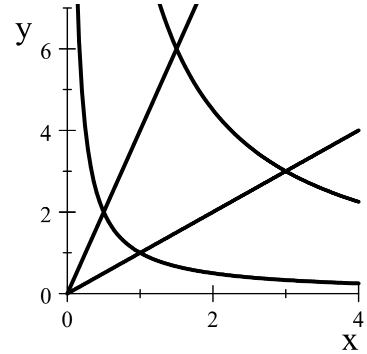
9. (20 points) Draw the region of integration and compute $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{y^3+1}\, dy\, dx$ by reversing the order of integration.

10. (20 points) Consider the vector field $\vec{F} = (x^3, y^3, x^2 z + y^2 z)$. First compute $\vec{\nabla} \cdot \vec{F}$ in rectangular coordinates. Then convert $\vec{\nabla} \cdot \vec{F}$ into cylindrical coordinates. Finally, compute $\int\int\int \vec{\nabla} \cdot \vec{F}\, dV$ over the solid region below the cone $z = 4 - \sqrt{x^2+y^2}$ and above the xy-plane.

11. (20 points) Compute $\iint_R x^2 \, dx\, dy$ over the diamond shaped region R bounded by

$$xy = 1, \qquad xy = 9, \qquad y = x, \qquad y = 4x$$

HINT: Use the curvilinear coordinates (u, v) where $x = uv$ and $y = \frac{u}{v}$.



a. (4 pts) What are the boundaries in terms of u and v?

b. (6 pts) Find the Jacobian factor $J = \left| \frac{\partial(x,y)}{\partial(u,v)} \right|$.

c. (3 pts) Express the integrand, x^2, in terms of u and v.

d. (7 pts) Compute the integral.