Name________________________

1-9	/45	11	/20
10	/20	12	/20
		Total	/105

MATH 251 Final Exam Version A Fall 2020

Sections 517/519 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^2 \int_0^z \int_0^{xz} 30x\,dy\,dx\,dz$.

a. 4

b. 8

c. 16

d. 32

e. 64

2. Find the center of mass of the quarter circle $x^2+y^2 \le 9$ in the first quadrant, if the density is $\delta = \sqrt{x^2+y^2}$.

a. $(\bar{x},\bar{y}) = \left(\frac{9}{4},\frac{9}{4}\right)$

b. $(\bar{x},\bar{y}) = \left(\frac{9}{2},\frac{9}{2}\right)$

c. $(\bar{x},\bar{y}) = \left(\frac{2}{9},\frac{2}{9}\right)$

d. $(\bar{x},\bar{y}) = \left(\frac{9}{2\pi},\frac{9}{2\pi}\right)$

e. $(\bar{x},\bar{y}) = \left(\frac{2\pi}{9},\frac{2\pi}{9}\right)$

3. The temperature in an ideal gas is given by $T = \kappa\frac{P}{\delta}$ where κ is a constant, P is the pressure and δ is the density. At a certain point $Q = (3,2,1)$, we have

$$P(Q) = 8 \qquad \vec{\nabla}P(Q) = (4,-2,-4)$$
$$\delta(Q) = 2 \qquad \vec{\nabla}\delta(Q) = (-1,4,2)$$

So at the point Q, the temperature is $T(Q) = 4\kappa$ and its gradient is $\vec{\nabla}T(Q) =$

a. $\kappa(-8.5,6,9)$

b. $\kappa(4,-9,-6)$

c. $\kappa(3,2,-2)$

d. $\kappa\left(\frac{1}{2},2\right)$

e. $\kappa\left(-\frac{1}{2},2\right)$

4. Compute $\iint_C e^{-x^2-y^2}\,dx\,dy$ over the disk enclosed in the circle $x^2+y^2=4$.

a. $\frac{\pi}{2}(1-e^{-4})$

b. $\pi(1-e^{-4})$

c. $\frac{\pi}{2}e^{-4}$

d. πe^{-4}

e. $2\pi e^{-4}$

5. Find the volume below $z=xy$ above the region between the curves $y=3x$ and $y=x^2$.

a. $\frac{81}{2}$

b. $\frac{81}{4}$

c. $\frac{81}{8}$

d. $\frac{243}{2}$

e. $\frac{243}{8}$

6. Compute the line integral $\int_P \vec{F}\cdot d\vec{s}$ for the vector field $\vec{F}=\langle y,x\rangle$ along the parabola $y=x^2$ from $x=-1$ to $x=2$.
HINT: Find a scalar potential.

a. 9

b. 7

c. 5

d. 3

e. 1

7. Compute $\iint \frac{1}{y}\, dS$ on the parametric surface $\vec{R}(u,v) = (u^2 - v^2, u^2 + v^2, 2uv)$

for $1 \le u \le 3$ and $1 \le v \le 4$.
HINT: Find the normal vector.

a. $6\sqrt{2}$

b. $12\sqrt{2}$

c. $24\sqrt{2}$

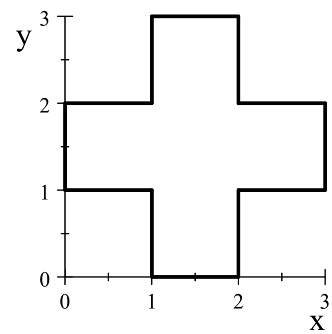
d. $64\sqrt{2}$

e. $272\sqrt{2}$

8. Compute $\oint (2x\sin y - 5y)\, dx + (x^2 \cos y - 4x)\, dy$

counterclockwise around the cross shown.

HINT: Use Green's Theorem.



a. -45

b. -10

c. 5

d. 10

e. 45

9. Compute $\iint \vec{F} \cdot d\vec{S}$ for $\vec{F} = (xy^2, yx^2, z(x^2 + y^2))$

over the complete surface of the solid

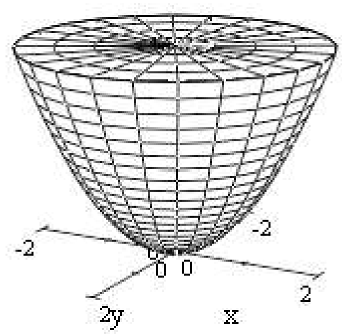
above the paraboloid $z = x^2 + y^2$

below the plane $z = 4$, oriented outward.

Note: The paraboloid may be parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$$

Hint: Use a Theorem.



a. $\frac{64}{5}\pi$

b. $\frac{64}{3}\pi$

c. $\frac{64}{15}\pi$

d. $\frac{256}{15}\pi$

e. $\frac{256}{3}\pi$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 points) Consider the surface which is the graph of the equation $xy - z^2 = 5$.

Letter the parts. Box your answers.

a. Find the normal vector to the surface at the point $P = (3,2,1)$.

b. Find the standard equation of the tangent plane to the surface at the point $P = (3,2,1)$. Then find its z-intercept.

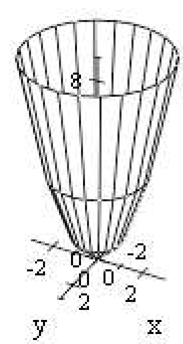
c. Find the parametric equation of the normal line to the surface at the point $P = (3,2,1)$. Then find where the normal line intersects the xy-plane.

11. (20 points) Find the point in the first octant on the graph of $z = \dfrac{8}{x^2 y}$ closest to the origin.

What is its distance from the origin? Box your answers.

12. (20 points) Verify Stokes' Theorem $\iint\limits_{P} \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint\limits_{\partial P} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = \langle -yz, xz, z^2 \rangle$ and the paraboloid $z = x^2 + y^2$ for $z \le 9$ oriented down and out.



Be sure to check and explain the orientations. Use the following steps.

Letter the parts. Box your answers.

LHS:

a. Compute the curl $\vec{\nabla} \times \vec{F}$ in rectangular coordinates.

b. The paraboloid surface, P, may be paranmetrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

What is $\vec{\nabla} \times \vec{F}$ on the paraboloid?

c. Find the normal to the paraboloid.

d. Compute the integral $\iint\limits_{P} \vec{\nabla} \times \vec{F} \cdot d\vec{S}$.

RHS:

e. The circle, C, at the top of the paraboloid may be parametrized by $\vec{r}(\theta) = (3\cos\theta, 3\sin\theta, 9)$.

What is $\vec{F}$ on the circle?

f. What is the tangent vector to the circle?

g. Compute the integral $\oint\limits_{\partial P} \vec{F} \cdot d\vec{s}$.