Name____________________________

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| 1-9 | /45 | 11 | /20 |
| 10 | /20 | 12 | /20 |
| | | Total | /105 |

MATH 251 Final Exam Version B Fall 2020

Sections 517/519 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

**1**. Compute $\int_1^2 \int_{1/y}^1 ye^{xy}\,dx\,dy$.

**a**. $e^2 - 2e$

**b**. $e^2 - e$

**c**. $e^2 - 2e - 1$

**d**. $e^2 - e - 1$

**e**. $e^2 - 2$

**2**. Find the center of mass of the half circle $x^2 + y^2 \le 9$ with $y \ge 0$,

if the density is $\delta = \sqrt{x^2 + y^2}$.

**a**. $(\bar{x},\bar{y}) = \left(0, \frac{9}{4}\right)$

**b**. $(\bar{x},\bar{y}) = \left(0, \frac{2}{9}\right)$

**c**. $(\bar{x},\bar{y}) = \left(0, \frac{9}{2}\right)$

**d**. $(\bar{x},\bar{y}) = \left(0, \frac{2\pi}{9}\right)$

**e**. $(\bar{x},\bar{y}) = \left(0, \frac{9}{2\pi}\right)$

**3**. Ham Deut is flying the Millennium Eagle through a dangerous zenithon field whose density is $\rho = xyz$. If his current position is $(x,y,z) = (1,-1,2)$, in what **unit** vector direction should he travel to **decrease** the density as fast as possible?

**a**. $(2,-2,1)$

**b**. $\left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

**c**. $(-2,2,-1)$

**d**. $\left(\frac{-2}{3}, \frac{2}{3}, \frac{-1}{3}\right)$

**e**. $\left(\frac{-2}{3}, \frac{-2}{3}, \frac{-1}{3}\right)$

**4**. Compute $\int_0^8 \int_{x^{1/3}}^2 \cos(y^2)\, dy\, dx$

HINT: Reverse the order of integration.

**a**. $\frac{1}{4}\sin(4) - \frac{1}{4}$

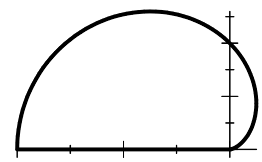
**b**. $\frac{1}{4}\sin(16) - \frac{1}{4}$

**c**. $\frac{1}{4}\sin(16)$

**d**. $\frac{1}{4}\sin(64) - \frac{1}{4}$

**e**. $\frac{1}{4}\sin(64)$

**5**. Find the volume below $z = y$ above the region between the $x$-axis and the upper half of the cardioid $r = 1 - \cos\theta$.



**a**. $\frac{1}{12}$

**b**. $\frac{1}{6}$

**c**. $\frac{2}{3}$

**d**. $\frac{4}{3}$

**e**. $\frac{8}{3}$

**6**. Compute the line integral $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (2x, 2y, 2z)$ along the curve $\vec{r}(t) = \left(\frac{2}{t}, \frac{4}{t}, \frac{6}{t}\right)$ from $(2,4,6)$ to $(1,2,3)$.

HINT: Find a scalar potential.

**a**. $-70$

**b**. $-42$

**c**. $0$

**d**. $42$

**e**. $70$

**7.** Find the area of the piece of the surface $z = xy$ above the semicircle $x^2 + y^2 \le 9$ for $y \ge 0$. Parametrize the surface as $\vec{R}(u,v) = (u, v, uv)$.
HINT: Find the normal vector.

**a.** $\frac{\pi}{3}(10^{3/2} - 1)$

**b.** $\frac{2\pi}{3}(10^{3/2} - 1)$

**c.** $9\pi$

**d.** $18\pi$

**e.** $36\pi$

**8.** Compute $\oint \vec{F} \cdot d\vec{s} = \oint P\,dx + Q\,dy$ for $\vec{F} = (P,Q) = (\sec(x^3) - 5y, \cos(y^5) + 3x)$ counterclockwise around the triangle with vertices $(0,0)$, $(8,0)$ and $(0,4)$.
Hint: Use Green's Theorem.

**a.** 12

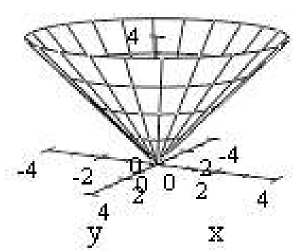
**b.** 16

**c.** 32

**d.** 64

**e.** 128

**9.** Compute $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the cone $z = \sqrt{x^2 + y^2}$ for $z \le 4$ oriented down and out for $\vec{F} = (y\sqrt{z}, -x\sqrt{z}, \sqrt{z})$.
Note: The cone may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.
Hint: Use a Theorem.




**a.** 4

**b.** $8\pi$

**c.** 16

**d.** 32

**e.** $64\pi$

Work Out: (Points indicated. Part credit possible. Show all work.)

**10**. (20 points ) Consider the surface which is the graph of the equation $xy - xz + yz = 11$.

Letter the parts. Box your answers.

**a**. Find the normal vector to the surface at the point $P = (3, 2, 1)$.

**b**. Find the standard equation of the tangent plane to the surface at the point $P = (3, 2, 1)$. Then find its $z$-intercept.

**c**. Find the parametric equation of the normal line to the surface at the point $P = (3, 2, 1)$. Then find where the normal line intersects the $xy$-plane.

**11**. (20 points ) The temperature around a candle is given by $T = 110 - x^2 - y^2 - 2z^2$.

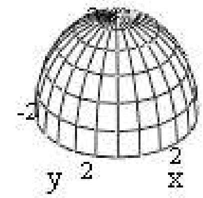
Find the maximum temperature on the plane $4x + 6y + 8z = 42$ and the point where it occur.

Box your answers.

**12**. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$

for the vector field $\vec{F} = \langle xz^2, yz^2, x^2 + y^2 \rangle$ and

the volume inside the hemisphere $H$: $0 \le z \le \sqrt{4 - x^2 - y^2}$




Be sure to check and explain the orientations. Use the following steps.

Letter the parts. Box your answers.

**LHS**:

**a**. Compute the divergence $\vec{\nabla} \cdot \vec{F}$ in rectangular coordinates.

**b**. What coordinate system will you use to compute the integral $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$ ?

What is $\vec{\nabla} \cdot \vec{F}$ in those coordinates?

What is $dV$ in those coordinates?

**c**. Compute the integral $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$.

**RHS**:

**d**. The disk at the bottom, $D$, may be parametrized as $\vec{R} = (r\cos\theta, r\sin\theta, 0)$.
What is $\vec{F}$ on the disk?

**e**. Find the normal to the disk.

**f**. Compute $\iint\limits_D \vec{F} \cdot d\vec{S}$

**g**. The hemisphere, $H$, may be parametrize as $\vec{R}(\varphi, \theta) = (2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi)$.
What is $\vec{F}$ on the hemisphere?

**h**. Find the normal to the hemisphere.

**i**. Compute $\iint\limits_H \vec{F} \cdot d\vec{S}$. HINT: What is $\int_0^{\pi/2} \cos^n\varphi \sin\varphi\, d\varphi$ ?

**j**. Combine $\iint\limits_D \vec{F} \cdot d\vec{S}$ and $\iint\limits_H \vec{F} \cdot d\vec{S}$ to get $\iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$.