Math 311-503 February 13, 2007

Test 1

Problem 1 (30 pts.) Let Π be the plane in $\mathbb{R}^3$ passing through the points $(1,0,0)$, $(0,0,1)$, and $(0,1,2)$. Let ℓ be the line in $\mathbb{R}^3$ passing through the points $(1,0,1)$ and $(-2,0,-2)$.

(i) Find a parametric representation for the line ℓ.
(ii) Find a parametric representation for the plane Π.
(iii) Find the point where the line ℓ intersects the plane Π.
(iv) Determine whether the plane $2x + y + 2z = 9$ is parallel to the plane Π.
(v) Find the angle between the line ℓ and the plane $2x + y + 2z = 9$.
(vi) Find the distance from the origin to the plane $2x + y + 2z = 9$.

Problem 2 (20 pts.) Find a quadratic polynomial $p(x)$ such that $p(1) = 1$, $p(2) = 3$, and $p(3) = 7$.

Problem 3 (20 pts.) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Compute the matrices A^2, A^3, and $q(A)$, where $q(x) = 2x^2 - 3x + 2$.

Problem 4 (30 pts.) Let $B = \begin{pmatrix} 0 & 5 & -1 & 0 \\ 0 & 3 & 0 & 2 \\ 1 & -3 & 4 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

(i) Evaluate the determinant of the matrix B.
(ii) Find the inverse matrix B^{-1}.

Bonus Problem 5 (25 pts.) Let P be the parallelogram bounded by the following two pairs of parallel lines in $\mathbb{R}^2$: $x + y = 1$, $x + y = 2$, $2x + 3y = 0$, and $2x + 3y = 5$.

(i) Find the vertices of P.
(ii) Find the angles of P.
(iii) Find the area of P.