

Homework assignment #4

Problem 1. Let $\mathbf{0}$ be the zero vector of a vector space V . Prove that $\beta\mathbf{0} = \mathbf{0}$ for each scalar β .

Problem 2. Consider the set $V = \mathbb{R}^2$ with addition \oplus and scalar multiplication \odot defined by

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &= (x_1 + x_2, y_1 + y_2), \\ \alpha \odot (x_1, x_2) &= (\alpha x_1, x_2).\end{aligned}$$

Is V a vector space with these operations? Justify your answer.

Problem 3. Determine whether the following sets form subspaces of the vector space \mathbb{R}^2 .

- (i) The set S_1 of all vectors $(x_1, x_2) \in \mathbb{R}^2$ such that $x_1 x_2 = 0$.
- (ii) The set S_2 of all vectors $(x_1, x_2) \in \mathbb{R}^2$ such that $x_1 = 3x_2$.

Problem 4. Determine whether the following vectors form a spanning set for the vector space \mathbb{R}^2 :

$$\text{(i) } \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 4 \\ 6 \end{pmatrix}; \quad \text{(ii) } \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Problem 5. Let $\mathbf{v} = (-1, 2, 3)$ and $\mathbf{w} = (3, 4, 2)$. Determine whether the following vectors belong to $\text{Span}(\mathbf{v}, \mathbf{w})$, the span of \mathbf{v} and \mathbf{w} :

$$\text{(i) } (2, 6, 6), \quad \text{(ii) } (-9, -2, 5).$$

Problem 6. Determine whether $\{x + 2, x + 1, x^2 - 1\}$ is a spanning set for \mathcal{P}_3 , the vector space of polynomials of degree at most 2.

Problem 7. Suppose that U_1 and U_2 are subspaces of the same vector space V . Prove that their intersection $U_1 \cap U_2$ is also a subspace of V .