MATH 323–502 Fall 2022

Homework assignment #9

Problem 1. Find the eigenvalues and the corresponding eigenspaces for each of the following matrices:

$$\textbf{(i)}\ \begin{pmatrix} 6 & -4 \\ 3 & -1 \end{pmatrix}, \qquad \textbf{(ii)}\ \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}, \qquad \textbf{(iii)}\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \textbf{(iv)}\ \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{pmatrix}.$$

Problem 2. A square matrix A is called *idempotent* if $A^2 = A$. Show that if λ is an eigenvalue of an idempotent matrix, then $\lambda = 0$ or $\lambda = 1$.

Problem 3. Factor each of the following matrices into a product XDX^{-1}, where D is diagonal:

$$\textbf{(i)}\ \begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}, \qquad \textbf{(ii)}\ \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}, \qquad \textbf{(iii)}\ \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}.$$

Problem 4. Let $A = \begin{pmatrix} 9 & -5 & 3 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{pmatrix}$. Find a matrix B such that $B^2 = A$.

Problem 5. Let A be a diagonalizable matrix whose eigenvalues are all either 1 or -1. Show that $A^{-1} = A$.