Homework assignment #5

Problem 1. Determine whether the following vectors are linearly independent in $\mathbb{R}^3$:
(i) $(2,1,-2)$, $(3,2,-2)$ and $(2,2,0)$;
(ii) $(2,1,-2)$, $(-2,-1,2)$ and $(4,2,-4)$;
(iii) $(1,1,3)$ and $(0,2,1)$.

Problem 2. Determine whether the following matrices are linearly independent in the vector space $\mathcal{M}_{2,2}(\mathbb{R})$ of 2×2 matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}.$$

Problem 3. Determine whether the following polynomials are linearly independent in the vector space $\mathcal{P}$ of polynomials:
(i) 2, x^2, x and $2x+3$; **(ii)** $x+2$, $x+1$ and x^2-1.

Problem 4. Show that the following functions are linearly independent in the vector space $C[0,1]$:
(i) e^x, e^{-x} and e^{2x}; **(ii)** 1, e^x+e^{-x} and e^x-e^{-x}.

Problem 5. The functions $f(x)=2x$ and $g(x)=|x|$ can be considered elements of the vector space $C[a,b]$ for any interval $[a,b]\subset\mathbb{R}$. Show that $f(x)$ and $g(x)$ are linearly independent in $C[-1,1]$ while being linearly dependent in $C[0,1]$.

Problem 6. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are linearly independent vectors in a vector space V. Prove that the vectors $\mathbf{w}_1=\mathbf{v}_1+\mathbf{v}_2$, $\mathbf{w}_2=\mathbf{v}_2+\mathbf{v}_3$ and $\mathbf{w}_3=\mathbf{v}_3+\mathbf{v}_1$ are also linearly independent in V.