MATH 323 Fall 2023

## Homework assignment #7

**Problem 1.** Vectors  $\mathbf{v}_1 = (1,1)$  and  $\mathbf{v}_2 = (-1,1)$  form a basis for the vector space  $\mathbb{R}^2$ . Vectors  $\mathbf{u}_1 = (5,3)$  and  $\mathbf{u}_2 = (3,2)$  form another basis for  $\mathbb{R}^2$ .

- (i) Find the transition matrix from the ordered basis  $\mathbf{v}_1, \mathbf{v}_2$  to the standard basis  $\mathbf{e}_1, \mathbf{e}_2$  and the transition matrix from  $\mathbf{e}_1, \mathbf{e}_2$  to  $\mathbf{v}_1, \mathbf{v}_2$ .
- (ii) Find the transition matrix from the ordered basis  $\mathbf{u}_1, \mathbf{u}_2$  to the standard basis  $\mathbf{e}_1, \mathbf{e}_2$  and the transition matrix from  $\mathbf{e}_1, \mathbf{e}_2$  to  $\mathbf{u}_1, \mathbf{u}_2$ .
- (iii) Find coordinates of the vector  $\mathbf{w} = (10, 7)$  relative to the basis  $\mathbf{v}_1, \mathbf{v}_2$  and coordinates of  $\mathbf{w}$  relative to the basis  $\mathbf{u}_1, \mathbf{u}_2$ .

**Problem 2.** Vectors  $\mathbf{v}_1 = (4,6,7)$ ,  $\mathbf{v}_2 = (0,1,1)$  and  $\mathbf{v}_3 = (0,1,2)$  form a basis for the vector space  $\mathbb{R}^3$ . Vectors  $\mathbf{u}_1 = (1,1,1)$ ,  $\mathbf{u}_2 = (1,2,2)$  and  $\mathbf{u}_3 = (2,3,4)$  form another basis for  $\mathbb{R}^3$ .

- (i) Find the transition matrix from the standard basis  $e_1$ ,  $e_2$ ,  $e_3$  to the ordered basis  $u_1$ ,  $u_2$ ,  $u_3$ .
- (ii) Find the transition matrix from the ordered basis  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  to the ordered basis  $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ .
- (iii) Find coordinates of the vector  $\mathbf{w} = 2\mathbf{v}_1 + 3\mathbf{v}_2 4\mathbf{v}_3$  relative to the basis  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ , coordinates of  $\mathbf{w}$  relative to the basis  $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ , and coordinates of  $\mathbf{w}$  relative to the standard basis.

**Problem 3.** Given

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad \qquad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad \qquad U = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix},$$

find vectors  $\mathbf{w}_1$  and  $\mathbf{w}_2$  so that U will be the transition matrix from the ordered basis  $\mathbf{w}_1, \mathbf{w}_2$  for  $\mathbb{R}^2$  to the ordered basis  $\mathbf{v}_1, \mathbf{v}_2$ .

**Problem 4.** Polynomials  $p_1(x) = 1$ ,  $p_2(x) = x$  and  $p_3(x) = x^2$  form a basis for the vector space  $\mathcal{P}_3$ . Polynomials  $q_1(x) = 1$ ,  $q_2(x) = 1 + x$  and  $q_3(x) = 1 + x + x^2$  form another basis for  $\mathcal{P}_3$ .

- (i) Find the transition matrix from the ordered basis  $q_1, q_2, q_3$  to the ordered basis  $p_1, p_2, p_3$ .
- (ii) Find the transition matrix from the ordered basis  $p_1, p_2, p_3$  to the ordered basis  $q_1, q_2, q_3$ .
- (iii) Find coordinates of the polynomial  $r(x) = 2x^2 + 3x 1$  relative to the ordered basis  $q_1, q_2, q_3$ .