Sample problems for Test 2 (page 1 of 2)

Any problem may be altered!

Problem I For each of the following sets $E \subset \mathbb{R}$ determine: the interior $\operatorname{int}(E)$, the exterior $\operatorname{int}(\mathbb{R} \setminus E)$, the boundary ∂E, the closure $\overline{E}$, and all isolated points. Is the set E open? Is it closed? Is it compact?

1. $E = \mathbb{N}$.
2. $E = \mathbb{Z}$.
3. $E = \mathbb{Q}$.
4. $E = \mathbb{R} \setminus \mathbb{Q}$.
5. $E = \mathbb{R}$.
6. $E = \emptyset$, the empty set.
7. $E = [0, 1)$.
8. $E = (0, \infty)$.
9. $E = [0, 1] \cup (2, 3)$.
10. $E = (-\infty, -1] \cup [1, \infty)$.
11. $E = \mathbb{R} \setminus \mathbb{Z}$.
12. $E = \bigcup_{n=1}^{\infty} (1/(n+1), 1/n)$.
13. $E = \{-1, 0, 1/2, 3\}$.
14. $E = \{1/n \mid n \in \mathbb{N}\}$.
15. $E = (-\infty, 0] \cup \{1/n \mid n \in \mathbb{N}\}$.
16. $E = \{(-1)^n(1 + n^{-1}) \mid n \in \mathbb{N}\}$.
17. $E = \{\log n \mid n \in \mathbb{N}\}$.
18. $E = \{\arctan n \mid n \in \mathbb{Z}\}$.
19. $E = \{2^q \mid q \in \mathbb{Q}\}$.
20. $E = \{p/q \mid p \text{ and } q \text{ are odd integers}\}$.

Problem II Find the following limits of functions (or show that the limit does not exist). Briefly explain.

21. $\lim\limits_{x\to\infty} \dfrac{(2x+3)^3\,(3x-2)^2}{x^5+5}$.
22. $\lim\limits_{x\to\infty} \dfrac{\sqrt[3]{x^2+1}}{x+1}$.
23. $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$.
24. $\lim\limits_{x\to 1} \dfrac{x^3-3x+2}{x^4-4x+3}$.
25. $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right)$.
26. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$.
27. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$.
28. $\lim\limits_{x\to 4} \dfrac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$.
29. $\lim\limits_{x\to\infty} (\sqrt{x(x+2)} - x)$.
30. $\lim\limits_{x\to\infty} x(\sqrt{x^2+1} - x)$.
31. $\lim\limits_{x\to 0} \dfrac{\tan 2x}{x}$.
32. $\lim\limits_{x\to 0} \dfrac{x - \sin 2x}{x + \sin 3x}$.
33. $\lim\limits_{x\to\infty} \dfrac{x - \sin 2x}{x + \sin 3x}$.
34. $\lim\limits_{x\to\infty} x \sin \dfrac{1}{x}$.
35. $\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$.
36. $\lim\limits_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$.
37. $\lim\limits_{x\to 0} \dfrac{1-\sqrt{\cos x}}{x^2}$.
38. $\lim\limits_{x\to 0} \arcsin x \cot x$.
39. $\lim\limits_{x\to \pi/2} \dfrac{\tan 3x}{\tan x}$.
40. $\lim\limits_{x\to 0} (1 + \sin x)^{1/x}$.
41. $\lim\limits_{x\to 0+} x^{\sin x}$.
42. $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+1}\right)^x$.
43. $\lim\limits_{x\to\infty} (1+x)^{1/x}$.
44. $\lim\limits_{x\to 0} (1+x^2)^{1/x}$.
45. $\lim\limits_{x\to 0+} x^x$.
46. $\lim\limits_{x\to\infty} \big(\log(2x+1) - \log x\big)$.
47. $\lim\limits_{x\to\infty} x\big(\log(x+1) - \log x\big)$.
48. $\lim\limits_{x\to\infty} \dfrac{\log(1+e^x)}{x}$.
49. $\lim\limits_{x\to-\infty} \dfrac{\log(1+e^x)}{x}$.
50. $\lim\limits_{x\to 0+} \log(1+x)\,\log x$.

Sample problems for Test 2 (page 2 of 2)

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Problem III For each of the following functions $f : E \to \mathbb{R}$ determine: points of discontinuity (and their types), points where f is not differentiable (do one-sided derivatives exist at those points? Is the derivative infinite?), extremal points, and intervals of decrease and increase. Finally, sketch the graph of f.

51. $f(x) = 1$ for $x \geq 0$, $f(x) = 0$ for $x < 0$.

52. $f(x) = |x|$, $x \in \mathbb{R}$.

53. $f(x) = x^3 - 3x$, $x \in \mathbb{R}$.

54. $f(x) = 2|x| - x^2$, $x \in \mathbb{R}$.

55. $f(0) = 0$, $f(x) = x^2 + \dfrac{2}{x}$ for $x \neq 0$.

56. $f(x) = \sqrt[3]{x}$, $x \in \mathbb{R}$.

57. $f(x) = \sqrt{x} + \sqrt{4 - x}$, $0 \leq x \leq 4$.

58. $f(x) = \sqrt[3]{x+1} - \sqrt[3]{x-1}$, $x \in \mathbb{R}$.

59. $f(x) = \sin x + \cos x$, $x \in \mathbb{R}$.

60. $f(x) = \cos x - \cos^2 x$, $x \in \mathbb{R}$.

61. $f(-\pi) = f(\pi) = 0$, $f(x) = \dfrac{1}{1 + \cos x}$ for $x \in (-\pi, \pi)$.

62. $f(x) = x + \sin x$, $x \in \mathbb{R}$.

63. $f(0) = 0$, $f(x) = x \arctan \dfrac{1}{x}$ for $x \neq 0$.

64. $f(x) = e^{\sin x}$, $x \in \mathbb{R}$.

65. $f(0) = -\dfrac{\pi}{2}$, $f(x) = \arctan(\log x)$ for $x > 0$.

66. $f(x) = xe^{-x}$, $x \in \mathbb{R}$.

67. $f(x) = (x^2 + 2)e^{-x^2}$, $x \in \mathbb{R}$.

68. $f(0) = 0$, $f(x) = \dfrac{\log x}{\sqrt{x}}$ for $x > 0$.

69. $f(x) = \log(1 + e^{-x})$, $x \in \mathbb{R}$.

70. $f(0) = 0$, $f(x) = x^x$ for $x > 0$.