Homework assignment #1

(due Friday, September 8)

Problem 1. Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties ($K_0 = \text{const}$) with the following sources and boundary conditions:

(i) $Q = 0, \quad u(0,t) = u_0, \quad u(L,t) = 0;$

(ii) $Q = 0, \quad \dfrac{\partial u}{\partial x}(0,t) = 0, \quad u(L,t) = u_0;$

(iii) $Q = 0, \quad u(0,t) = u_0, \quad \dfrac{\partial u}{\partial x}(L,t) = \alpha;$

(iv) $Q/K_0 = 1, \quad u(0,t) = u_1, \quad u(L,t) = u_2;$

(v) $Q(x)/K_0 = x^2, \quad u(0,t) = u_0, \quad \dfrac{\partial u}{\partial x}(L,t) = 0;$

(vi) $Q = 0, \quad u(0,t) = u_0, \quad \dfrac{\partial u}{\partial x}(L,t) + u(L,t) = 0.$

Problem 2. Solve the initial value problem for the one-dimensional wave equation on the infinite segment:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t \geq 0$$

with the following initial conditions:

(i) $u(x,0) = \sin x, \quad \dfrac{\partial u}{\partial t}(x,0) = 0;$

(ii) $u(x,0) = 0, \quad \dfrac{\partial u}{\partial t}(x,0) = \cos x;$

(iii) $u(x,0) = \sin x, \quad \dfrac{\partial u}{\partial t}(x,0) = \cos x.$

Problem 3. Solve the following initial-boundary value problem for the one-dimensional wave equation on a semi-infinite segment:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad x \geq 0, \quad t \geq 0;$$

$$u(x,0) = \cos x, \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad x \geq 0;$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad t \geq 0.$$