Homework assignment #10
(due Friday, December 1)

Problem 1. Show that the functions $h_\epsilon(x) = \dfrac{1}{2\epsilon}\, e^{-|x|/\epsilon}$ ($\epsilon > 0$) form a delta family.

Problem 2. Let $g(x) = |x|$ for all $x \in \mathbb{R}$. Find the second derivative g'' in the sense of distributions.

Problem 3. Consider

$$\frac{d^2u}{dx^2} = f(x) \quad \text{with} \quad u(0) = 0 \quad \text{and} \quad \frac{du}{dx}(L) = 0.$$

(i) Solve by direct integration.
(ii) Determine $G(x, x_0)$ so that

$$u(x) = \int_0^L f(x_0) G(x, x_0)\, dx_0.$$

Problem 4. Consider

$$\frac{d^2G}{dx^2} = \delta(x - x_0) \quad \text{with} \quad G(0, x_0) = 0 \quad \text{and} \quad \frac{dG}{dx}(L, x_0) = 0.$$

(i) Solve directly.
(ii) Check whether $G(x, x_0) = G(x_0, x)$.
(iii) Compare with Problem 3.

Problem 5. (i) Solve

$$\frac{dG}{dx} + G = \delta(x - x_0) \quad \text{with} \quad G(0, x_0) = 0.$$

(ii) Show that $G(x, x_0)$ is not symmetric.
(iii) Solve

$$u' + u = f \quad \text{with} \quad u(0) = 0$$

for any function f on $[0, L]$.