Homework assignment #3
(due Friday, September 22)

Problem 1. For the following functions, sketch the Fourier series of $f(x)$ (on the interval $-L \le x \le L$) and determine the Fourier coefficients:

(i) $f(x) = e^{-x}$; (ii) $f(x) = \begin{cases} 1, & x \ge 0, \\ 0, & x < 0. \end{cases}$

Problem 2. For the following functions, sketch $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$:

(i) $f(x) = 1$;

(ii) $f(x) = 1 + x$;

(iii) $f(x) = \begin{cases} x, & x < 0, \\ 1 + x, & x \ge 0; \end{cases}$

(iv) $f(x) = e^{x}$;

(v) $f(x) = \begin{cases} 2, & x \le 0, \\ e^{-x}, & x > 0. \end{cases}$

Problem 3. Sketch the Fourier sine series of $f(x) = x$. Also, roughly sketch the sum of a (large) finite number of nonzero terms of the Fourier sine series.

Problem 4. (i) Consider a function $f(x)$ which is even around $x = L/2$. Show that the odd coefficients (n odd) of the Fourier cosine series of $f(x)$ on $0 \le x \le L$ are zero.

(ii) Explain the result of part (i) by considering a Fourier cosine series of $f(x)$ on the interval $0 \le x \le L/2$.

Problem 5. Fourier series can be defined on other intervals besides $-L \le x \le L$. Suppose that a smooth function $g(y)$ is defined for $a \le y \le b$. Represent $g(y)$ using periodic trigonometric functions with period $b - a$. Determine formulas for the coefficients.

Hint: use the linear transformation $y = \dfrac{a+b}{2} + \dfrac{b-a}{2L}x$.