Homework assignment #4
(due Friday, October 6)

Problem 1. Solve Laplace's equation inside a rectangle $0 \le x \le L$, $0 \le y \le H$, with the following boundary conditions:

$$\frac{\partial u}{\partial x}(0,y) = 0, \qquad \frac{\partial u}{\partial x}(L,y) = 0, \qquad u(x,0) = 0, \qquad u(x,H) = f(x).$$

Problem 2. Solve Laplace's equation inside a semicircle of radius a ($0 < r < a$, $0 < \theta < \pi$) subject to the boundary conditions: $u = 0$ on the diameter and $u(a,\theta) = g(\theta)$.

Problem 3. Solve Laplace's equation inside a 90^{o} sector of a circular annulus ($a < r < b$, $0 < \theta < \pi/2$) subject to the boundary conditions:

$$u(r,0) = 0, \qquad u(r,\pi/2) = 0, \qquad u(a,\theta) = 0, \qquad u(b,\theta) = f(\theta).$$

Problem 4. Consider the heat equation in a two-dimensional rectangular region, $0 < x < L$, $0 < y < H$,

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

subject to the initial condition $u(x,y,0) = f(x,y)$.

Solve the initial-boundary value problem and analyze the temperature as $t \to \infty$ if the boundary conditions are:

$$\frac{\partial u}{\partial x}(0,y,t) = 0, \qquad \frac{\partial u}{\partial x}(L,y,t) = 0, \qquad \frac{\partial u}{\partial y}(x,0,t) = 0, \qquad \frac{\partial u}{\partial y}(x,H,t) = 0.$$

Problem 5. Consider the wave equation for a vibrating rectangular membrane ($0 < x < L$, $0 < y < H$)

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

subject to the initial conditions $u(x,y,0) = 0$ and $\dfrac{\partial u}{\partial t}(x,y,0) = f(x,y)$.

Solve the initial-boundary value problem if

$$\frac{\partial u}{\partial x}(0,y,t) = 0, \qquad \frac{\partial u}{\partial x}(L,y,t) = 0, \qquad \frac{\partial u}{\partial y}(x,0,t) = 0, \qquad \frac{\partial u}{\partial y}(x,H,t) = 0.$$